

Formula Card for Johnson & Kuby, ELEMENTARY STATISTICS, Eleventh Edition

Sample mean:

$$\bar{x} = \frac{\sum x}{n} \quad (2.1)$$

Depth of sample median:

$$d(\tilde{x}) = (n + 1)/2 \quad (2.2)$$

Range: $H - L$

Sample variance:

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1} \quad (2.5)$$

or

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1} \quad (2.9)$$

Sample standard deviation:

$$s = \sqrt{s^2} \quad (2.6)$$

Chebyshev's theorem: at least $1 - (1/k^2)$ (p. 99)

Sum of squares of x:

$$SS(x) = \sum x^2 - ((\sum x)^2/n) \quad (2.8)$$

Sum of squares of y:

$$SS(y) = \sum y^2 - ((\sum y)^2/n) \quad (3.3)$$

Sum of squares of xy:

$$SS(xy) = \sum xy - ((\sum x \cdot \sum y)/n) \quad (3.4)$$

Pearson's correlation coefficient:

$$r = SS(xy) / \sqrt{SS(x) \cdot SS(y)} \quad (3.2)$$

Equation for line of best fit: $\hat{y} = b_0 + b_1x$ (p. 146)

Slope for line of best fit: $b_1 = SS(xy)/SS(x)$ (3.6)

y-intercept for line of best fit:

$$b_0 = [\sum y - (b_1 \cdot \sum x)]/n \quad (3.7)$$

Empirical (observed) probability:

$$P'(A) = n(A)/n \quad (4.1)$$

Theoretical probability for equally likely sample space:

$$P(A) = n(A)/n(S) \quad (4.2)$$

Complement rule:

$$P(\text{not } A) = P(\bar{A}) = 1 - P(A) \quad (4.3)$$

General addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad (4.4)$$

General multiplication rule:

$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad (4.5)$$

Special addition rule for mutually exclusive events:

$$P(A \text{ or } B \text{ or } \dots \text{ or } E) = P(A) + P(B) + \dots + P(E) \quad (4.6)$$

Special multiplication rule for independent events:

$$P(A \text{ and } B \text{ and } \dots \text{ and } E) = P(A) \cdot P(B) \cdot \dots \cdot P(E) \quad (4.7)$$

Mean of discrete random variable:

$$\mu = \sum [xP(x)] \quad (5.1)$$

Variance of discrete random variable:

$$\sigma^2 = \sum [x^2P(x)] - \{\sum [xP(x)]\}^2 \quad (5.3a)$$

Standard deviation of discrete random variable:

$$\sigma = \sqrt{\sigma^2} \quad (5.4)$$

Factorial: $n! = (n)(n-1)(n-2) \cdots 2 \cdot 1$ (p. 248)

Binomial coefficient:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \quad (5.6)$$

Binomial probability function:

$$P(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}, x = 0, 1, 2, \dots, n \quad (5.5)$$

Mean of binomial random variable: $\mu = np$ (5.7)

Standard deviation, binomial random variable:

$$\sigma = \sqrt{npq} \quad (5.8)$$

Standard score: $z = (x - \mu)/\sigma$ (6.3)

Standard score for $\bar{x}$: $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ (7.2)

Confidence interval for mean, μ (σ known):

$$\bar{x} \pm z(\alpha/2) \cdot (\sigma/\sqrt{n}) \quad (8.1)$$

Sample size for $1 - \alpha$ confidence estimate for μ :

$$n = [z(\alpha/2) \cdot \sigma/E]^2 \quad (8.3)$$

Calculated test statistic for $H_0: \mu = \mu_0$ (σ known):

$$z^* = (\bar{x} - \mu_0)/(\sigma/\sqrt{n}) \quad (8.4)$$

Confidence interval estimate for mean, μ (σ unknown):

$$\bar{x} \pm t(df, \alpha/2) \cdot (s/\sqrt{n}) \text{ with } df = n - 1 \quad (9.1)$$

Calculated test statistic for $H_0: \mu = \mu_0$ (σ unknown):

$$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \text{ with } df = n - 1 \quad (9.2)$$

Confidence interval estimate for proportion, p :

$$p' \pm z(\alpha/2) \cdot \sqrt{p'q'/n}, p' = x/n \quad (9.6)$$

Sample Size for $1 - \alpha$ Confidence Interval of p

$$n = \frac{[z(\alpha/2)]^2 p^* q^*}{E^2} \quad (9.8)$$

Calculated test statistic for $H_0: p = p_0$:

$$z^* = (p' - p_0)/\sqrt{p_0q_0/n}, p' = x/n \quad (9.9)$$

Calculated test statistic for $H_0: \sigma^2 = \sigma_0^2$ or $\sigma = \sigma_0$:

$$\chi^2 = (n-1)s^2/\sigma_0^2, df = n - 1 \quad (9.10)$$

Mean difference between two dependent samples:

$$\text{Paired difference: } d = x_1 - x_2 \quad (10.1)$$

Confidence interval for mean difference, μ_d :

$$\bar{d} \pm t(df, \alpha/2) \cdot s_d/\sqrt{n} \text{ with } df = n - 1 \quad (10.2)$$

Sample mean of paired differences:

$$\bar{d} = \sum d/n \quad (10.3)$$

Sample standard deviation of paired differences:

$$s_d = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}} \quad (10.4)$$

Calculated test statistic for $H_0: \mu_d = \mu_0$:

$$t^* = (\bar{d} - \mu_0) / (s_d / \sqrt{n}), \quad df = n - 1 \quad (10.5)$$

Difference between means of two independent samples:

Degrees of freedom:

$$df = \text{smaller of } (n_1 - 1) \text{ or } (n_2 - 1) \quad (\text{p. 496})$$

Confidence interval estimate for $\mu_1 - \mu_2$:

$$(\bar{x}_1 - \bar{x}_2) \pm t(df, \alpha/2) \sqrt{(s_1^2/n_1) + (s_2^2/n_2)} \quad (10.8)$$

Calculated test statistic for $H_0: \mu_1 - \mu_2 = (\mu_1 - \mu_2)_0$:

$$t^* = [(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0] / \sqrt{(s_1^2/n_1) + (s_2^2/n_2)} \quad (10.9)$$

Difference between proportions of two independent samples:

Confidence interval for $p_1 - p_2$:

$$(p'_1 - p'_2) \pm z(\alpha/2) \cdot \sqrt{\frac{p'_1 q'_1}{n_1} + \frac{p'_2 q'_2}{n_2}} \quad (10.11)$$

Pooled observed probability:

$$p'_p = (x_1 + x_2) / (n_1 + n_2) \quad (10.13)$$

$$q'_p = 1 - p'_p \quad (10.14)$$

Calculated test statistic for $H_0: p_1 - p_2 = 0$:

$$z^* = \frac{p'_1 - p'_2}{\sqrt{(p'_p)(q'_p) \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) \right]}} \quad (10.15)$$

Ratio of variances between two independent samples:

Calculated test statistic for $H_0: \sigma_1^2 / \sigma_2^2 = 1$:

$$F^* = s_1^2 / s_2^2 \quad (10.16)$$

Calculated test statistic for enumerative data:

$$\chi^2^* = \sum [(O - E)^2 / E] \quad (11.1)$$

Multinomial experiment:

$$\text{Degrees of freedom: } df = k - 1 \quad (11.2)$$

$$\text{Expected frequency: } E = n \cdot p \quad (11.3)$$

Test for independence or Test of homogeneity:

Degrees of freedom:

$$df = (r - 1) \cdot (c - 1) \quad (11.4)$$

$$\text{Expected value: } E = (R \cdot C) / n \quad (11.5)$$

Mathematical model:

$$x_{c,k} = \mu + F_c + \epsilon_{k(c)} \quad (12.13)$$

Total sum of squares:

$$SS(\text{total}) = \sum (x^2) - \frac{(\sum x)^2}{n} \quad (12.2)$$

Sum of squares due to factor:

$$\left[\left(\frac{C_1^2}{k_1} \right) + \left(\frac{C_2^2}{k_2} \right) + \left(\frac{C_3^2}{k_3} \right) + \dots \right] - \left[\frac{(\sum x)^2}{n} \right] \quad (12.3)$$

Sum of squares due to error:

$$SS(\text{error}) = \sum (x^2) - [(C_1^2/k_1) + (C_2^2/k_2) + (C_3^2/k_3) + \dots] \quad (12.4)$$

Degrees of freedom for total:

$$df(\text{total}) = n - 1 \quad (12.6)$$

Degrees of freedom for factor:

$$df(\text{factor}) = c - 1 \quad (12.5)$$

Degrees of freedom for error:

$$df(\text{error}) = n - c \quad (12.7)$$

Mean square for factor:

$$MS(\text{factor}) = SS(\text{factor}) / df(\text{factor}) \quad (12.10)$$

Mean square for error:

$$MS(\text{error}) = SS(\text{error}) / df(\text{error}) \quad (12.11)$$

Calculated test statistic for H_0 : Mean value is same at all levels:

$$F^* = MS(\text{factor}) / MS(\text{error}) \quad (12.12)$$

Covariance of x and y :

$$\text{covar}(x, y) = \sum [(x - \bar{x})(y - \bar{y})] / (n - 1) \quad (13.1)$$

Pearson's correlation coefficient:

$$r = \text{covar}(x, y) / (s_x \cdot s_y) \quad (13.2)$$

or

$$r = SS(xy) / \sqrt{SS(x) \cdot SS(y)} \quad (3.2) \text{ or } (13.3)$$

$$\text{Experimental error: } e = y - \hat{y} \quad (13.5)$$

$$\text{Estimated variance of error: } s_e^2 = \sum (y - \hat{y})^2 / (n - 2) \quad (13.6)$$

or

$$s_e^2 = \frac{(\sum y^2) - (b_0)(\sum y) - (b_1)(\sum xy)}{n - 2} \quad (13.8)$$

Standard deviation about the line of best fit:

$$s_e = \sqrt{s_e^2} \quad (13.9)$$

Estimate for variance of slope:

$$s_{b_1}^2 = \frac{s_e^2}{SS(x)} = \frac{s_e^2}{\sum x^2 - [(\sum x)^2 / n]} \quad (13.12)$$

Confidence interval for β_1 :

$$b_1 \pm t(df, \alpha/2) \cdot s_{b_1} \quad (13.14)$$

Calculated test statistic for $H_0: \beta_1 = 0$:

$$t^* = (b_1 - \beta_1) / s_{b_1} \text{ with } df = n - 2 \quad (13.15)$$

Confidence interval for mean value of y at x_0 :

$$\hat{y} \pm t(n - 2, \alpha/2) \cdot s_e \cdot \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS(x)}} \quad (13.17)$$

Prediction interval for y at x_0 :

$$\hat{y} \pm t(n - 2, \alpha/2) \cdot s_e \cdot \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS(x)}} \quad (13.16)$$

Mann-Whitney U test:

$$U_a = n_a \cdot n_b + [(n_b) \cdot (n_b + 1) / 2] - R_b \quad (14.3)$$

$$U_b = n_a \cdot n_b + [(n_a) \cdot (n_a + 1) / 2] - R_a \quad (14.4)$$

Spearman's rank correlation coefficient:

$$r_s = 1 - \left[\frac{6 \sum d^2}{n(n^2 - 1)} \right] \quad (14.11)$$

Table E The Standard Normal Distribution

Cumulative Standard Normal Distribution

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

For *z* values less than −3.49, use 0.0001.

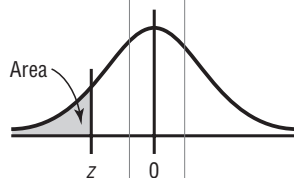


Table E (continued)

Cumulative Standard Normal Distribution

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For z values greater than 3.49, use 0.9999.

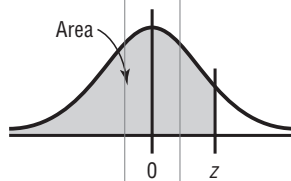


Table F	The t Distribution					
d.f.	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807
24		1.318	1.711	2.064	2.492	2.797
25		1.316	1.708	2.060	2.485	2.787
26		1.315	1.706	2.056	2.479	2.779
27		1.314	1.703	2.052	2.473	2.771
28		1.313	1.701	2.048	2.467	2.763
29		1.311	1.699	2.045	2.462	2.756
30		1.310	1.697	2.042	2.457	2.750
32		1.309	1.694	2.037	2.449	2.738
34		1.307	1.691	2.032	2.441	2.728
36		1.306	1.688	2.028	2.434	2.719
38		1.304	1.686	2.024	2.429	2.712
40		1.303	1.684	2.021	2.423	2.704
45		1.301	1.679	2.014	2.412	2.690
50		1.299	1.676	2.009	2.403	2.678
55		1.297	1.673	2.004	2.396	2.668
60		1.296	1.671	2.000	2.390	2.660
65		1.295	1.669	1.997	2.385	2.654
70		1.294	1.667	1.994	2.381	2.648
75		1.293	1.665	1.992	2.377	2.643
80		1.292	1.664	1.990	2.374	2.639
90		1.291	1.662	1.987	2.368	2.632
100		1.290	1.660	1.984	2.364	2.626
500		1.283	1.648	1.965	2.334	2.586
1000		1.282	1.646	1.962	2.330	2.581
(z) ∞		1.282 ^a	1.645 ^b	1.960	2.326 ^c	2.576 ^d

^aThis value has been rounded to 1.28 in the textbook.

^bThis value has been rounded to 1.65 in the textbook.

^cThis value has been rounded to 2.33 in the textbook.

^dThis value has been rounded to 2.58 in the textbook.

Source: Adapted from W. H. Beyer, *Handbook of Tables for Probability and Statistics*, 2nd ed., CRC Press, Boca Raton, Fla., 1986. Reprinted with permission.

