Chapter 8

Introduction to Statistical Inferences

Chapter 8 Overview

- 8-1 The Nature of Estimation
- 8-2 Estimation of Mean μ (σ Known)
- 8-3 The Nature of Hypothesis Testing
- 8-4 Hypothesis Test of Mean μ (σ Known): A Probability-Value Approach
- 8-5 Hypothesis Test of Mean μ (σ known): A Classical Approach

8.1 The Nature of Estimation

The central limit theorem gave us some very important information about the sampling distribution of sample means (SDSM).

Specifically, it stated that in many realistic cases (when the random sample is large enough) a distribution of sample means is normally or approximately normally distributed about the mean of the population.

With this information we were able to make probability statements about the likelihood of certain sample mean values occurring when samples are drawn from a population with a known mean and a known standard deviation.

8.1 The Nature of Estimation

- We are now ready to turn this situation around to the case in which the population mean is not known.

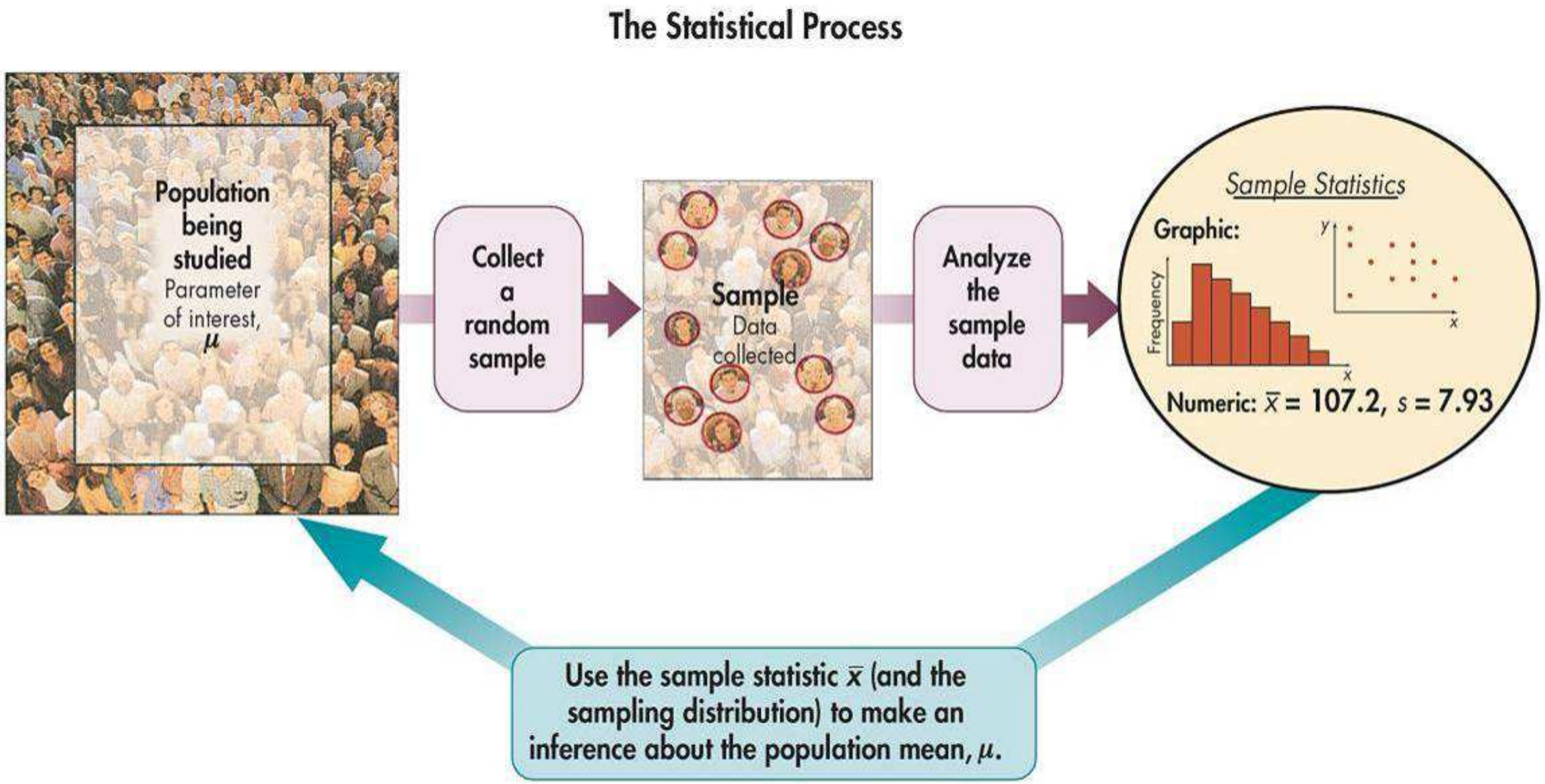
- We will draw one sample, calculate its mean value, and then make an inference about the value of the population mean based on the sample's mean value.

- The objective of inferential statistics is to use the information contained in the sample data to increase our knowledge of the sampled population.

- We will learn about making two types of inferences:
(1) estimating the value of a population parameter and
(2) testing a hypothesis.

8.1 The Nature of Estimation

- The sampling distribution of sample means (SDSM) is the key to making these inferences as shown in Figure 8.1.



Where the Sampling Distribution Fits into the Statistical Process

Figure 8.1

8.1 The Nature of Estimation

- In this chapter, we deal with questions about the population mean using two methods that assume the value of the population standard deviation is a known quantity.

- This assumption is seldom realized in real-life problems, but it will make our first look at the techniques of inference much simpler.

8-2 Estimation of Mean μ (σ Known)

- A point estimate is a specific numerical value estimate of a parameter.
- The best point estimate of the population mean μ is the sample mean $\bar{X}$.

Point Estimate for μ

- A machine produces parts with lengths that are normally distributed with $\sigma = 0.5$. A sample of 10 parts has a mean length of 75.92. Find the best point estimate for μ.

Point Estimate for μ

- Two hundred fish caught in Cayuga Lake had a mean length of 14.3 inches. The population standard deviation is 2.5 inches. Find a point estimate for μ.

Confidence Intervals for the Mean When σ Is Known and Sample Size

- An **interval estimate** of a parameter is an interval or a range of values used to estimate the parameter.
- This estimate may or may not contain the value of the parameter being estimated.

Interval Estimate for μ

- A machine produces parts with lengths that are normally distributed with $\sigma = 0.5$. A sample of 10 parts has a mean length of 75.92.
  - An interval estimate for this scenario is
    $75.552 < \mu < 76.288$
  - The actual value of μ is not necessarily in this interval. It is just a good estimate for where μ is located.

Confidence Level of an Interval Estimate

- The **confidence level** of an interval estimate of a parameter is the probability that the interval estimate will contain the parameter, assuming that a large number of samples are selected and that the estimation process on the same parameter is repeated.

Confidence Level

- A machine produces parts with lengths that are normally distributed with $\sigma = 0.5$. A sample of 10 parts has a mean length of 75.92.
  - Recall the interval estimate for this scenario was $75.552 < \mu < 76.288$
  - The confidence level of this interval estimate is 0.98 or 98%.
    - In other words, we have a 98% chance of the actual value of μ being in this interval range.

Confidence Interval

- A **confidence interval** is a specific interval estimate of a parameter determined by using data obtained from a sample and by using the specific confidence level of the estimate.
  - This is a specific type of interval estimate.

Confidence Interval

- A machine produces parts with lengths that are normally distributed with $\sigma = 0.5$. A sample of 10 parts has a mean length of 75.92.
  - Recall the interval estimate for this scenario was $75.552 < \mu < 76.288$
  - This is actually what is called a 98% confidence interval.
    - It was determined using the confidence level, 98%, the sample mean, $\bar{x}$, the population standard deviation, σ, and the sample size, n.

Formula for the Confidence Interval of the Mean for a Specific α

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

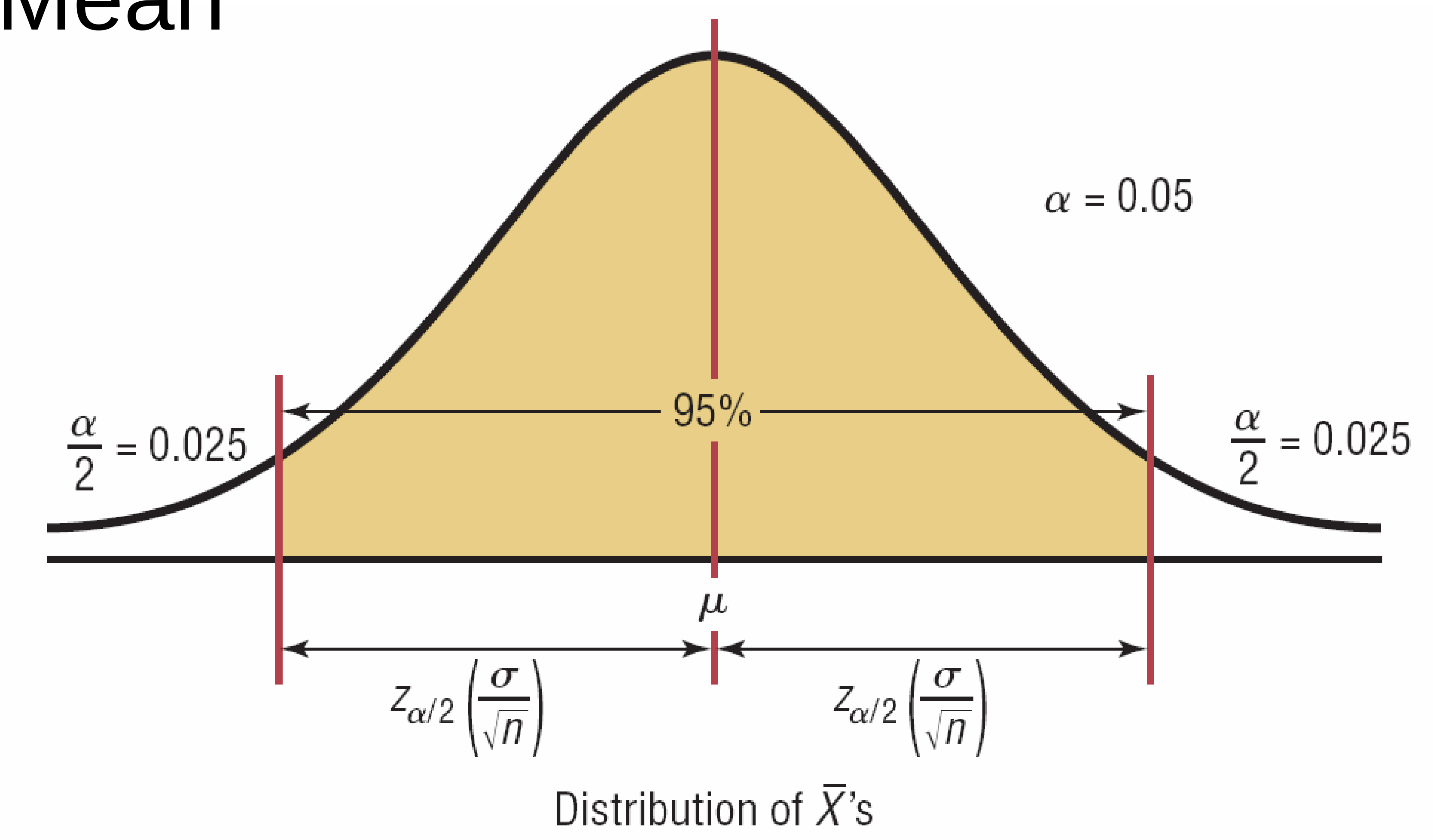
For a 90% confidence interval: $z_{\alpha/2} = 1.65$

For a 95% confidence interval: $z_{\alpha/2} = 1.96$

For a 98% confidence interval: $z_{\alpha/2} = 2.33$

For a 99% confidence interval: $z_{\alpha/2} = 2.58$

95% Confidence Interval of the Mean



Confidence Interval for a Mean

Rounding Rule

When you are computing a confidence interval for a population mean by using *raw data,* round off to one more decimal place than the number of decimal places in the original data.

When you are computing a confidence interval for a population mean by using a sample mean and a standard deviation, round off to the same number of decimal places as given for the mean.

Ex.) 1 – Confidence Interval for a Mean

Workers' Distractions A recent study showed that the modern working person experiences an average of 2.1 hours per day of distractions (phone calls, e-mails, impromptu visits, etc.). A random sample of 50 workers for a large corporation found that these workers were distracted an average of 1.8 hours per day and the population standard deviation was 20 minutes. Estimate the true mean population distraction time with 90% confidence, and compare your answer to the results of the study.

$$\overline{X} = 1.8, \sigma = \frac{20}{60}, n = 50, 90\% \rightarrow z = 1.65$$

Ex.) 1 – Confidence Interval for a Mean

$$\bar{X} = 1.8, \sigma = \frac{20}{60}, n = 50, 90\% \rightarrow z = 1.65$$

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$1.8 - 1.65\left(\frac{1/3}{\sqrt{50}}\right) < \mu < 1.8 + 1.65\left(\frac{1/3}{\sqrt{50}}\right)$$

$$1.8 - 0.08 < \mu < 1.8 + 0.08$$

$$1.72 < \mu < 1.88$$

We are 90% confident that the average amount of time people are distracted at work is between 1.72 and 1.88 hours. This estimate is lower than the recent study.

Ex.) 2 – Confidence Interval for a Mean

Actuary Exams A survey of 35 individuals who passed the seven exams and obtained the rank of Fellow in the actuarial field finds the average salary to be \$150,000. If the standard deviation for the population is \$15,000, construct a 95% confidence interval for all Fellows.

Class Activity

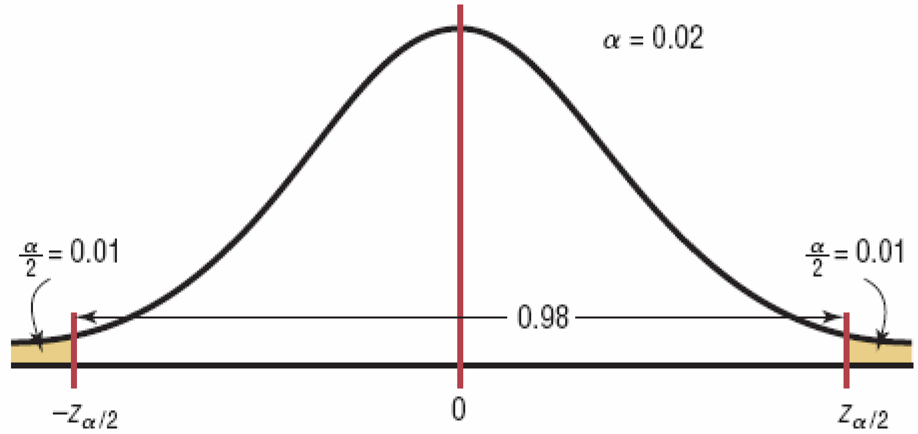
- Two hundred fish caught in Cayuga Lake had a mean length of 14.3 inches. The population standard deviation is 2.5 inches. Find the 99% confidence interval for the population mean length.

Maximum Error of the Estimate

The maximum error of the estimate is the maximum likely difference between the point estimate of a parameter and the actual value of the parameter.

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Finding $z_{\alpha/2}$ for 98% CL.



We want to look up the value for the area less than $z_{\alpha/2}$. That is 0.9900. The closest value on the table is 0.9901.

Table E
The Standard Normal Distribution

z	.00	.01	.02	.03	...	.09
0.0						
0.1						
⋮						
2.3				0.9901		

$$z_{\alpha/2} = 2.33$$

Ex.) 3 – Confidence Interval for a Mean

Number of Farms A random sample of the number of farms (in thousands) in various states follows. Estimate the mean number of farms per state with 90% confidence. Assume $\sigma = 31$.

47	95	54	33	64	4	8	57	9	80
8	90	3	49	4	44	79	80	48	16
68	7	15	21	52	6	78	109	40	50
29									

Ex.) 3 – Confidence Interval for a Mean

Step 1: Find the mean. $\bar{X} = 43.45$

Step 2: Find $\alpha/2$. 90% CL → $\alpha/2 = 0.05.$

Step 3: Find $z_{\alpha/2}$. 90% CL → $\alpha/2 = 0.05$ → $z_{.05} = 1.65$

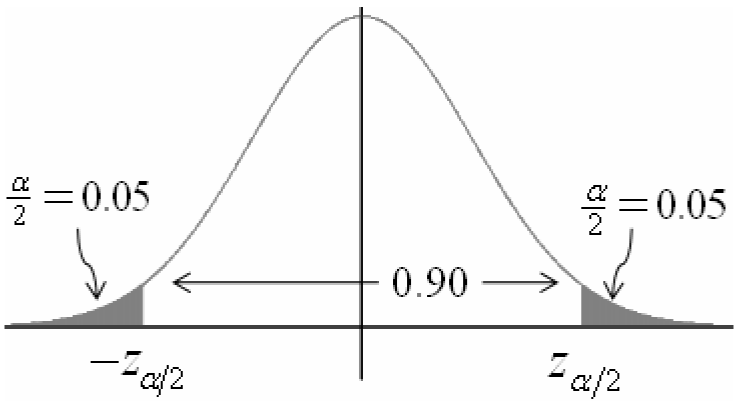


Table E

The Standard Normal Distribution

z	.00	…	.04	.05	…	.09
0.0						
0.1						
⋮						
1.6			0.9495	$\boxed{0.9505}$		

Ex.) 3 – Confidence Interval for a Mean

Step 4: Substitute in the formula.

$$\overline{X} = 43.45, \sigma = 31, n = 31, 90\% \rightarrow z = 1.65$$

$$\overline{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \overline{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$43.45 - 1.65\left(\frac{31}{\sqrt{31}}\right) < \mu < 43.45 + 1.65\left(\frac{31}{\sqrt{31}}\right)$$

$$43.45 - 9.2 < \mu < 43.45 + 9.2$$

$$34.3 < \mu < 52.7$$

One can be 90% confident that the population mean of the number of farms per state is between 34.3 thousand and 52.7 thousand, based on a sample of 31 states.

Technology Note

This chapter and subsequent chapters include examples using raw data. If you are using computer or calculator programs to find the solutions, the answers you get may vary somewhat from the ones given in the textbook.

This is so because computers and calculators do not round the answers in the intermediate steps and can use 12 or more decimal places for computation. Also, they use more exact values than those given in the tables in the back of this book.

These discrepancies are part and parcel of statistics.

Formula for Minimum Sample Size Needed for an Interval Estimate of the Population Mean

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

where *E* is the maximum error of estimate. If necessary, round the answer up to obtain a whole number. That is, if there is any fraction or decimal portion in the answer, use the next whole number for sample size *n.*

Ex.) 4 – Confidence Interval for a Mean

Time on Homework A university dean of students wishes to estimate the average number of hours students spend doing homework per week. The standard deviation from a previous study is 6.2 hours. How large a sample must be selected if he wants to be 99% confident of finding whether the true mean differs from the sample mean by 1.5 hours?

$$99\% \rightarrow z = 2.58, E = 1.5, \sigma = 6.2$$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left(\frac{2.58 \cdot 6.2}{1.5} \right)^2 = 113.7 \approx 114$$

Therefore, to be 99% confident that the estimate is within 1.5 hours of the true mean, the dean needs at least a sample of 114 students.

Finding Sample Size

- A high tech company wants to estimate the mean number of years of college education its employees have completed. A good estimate of the standard deviation for the number of years of college is 1.0. How large a sample needs to be taken to estimate μ to within 0.5 of a year with 99% confidence?

Class Activity: Confidence Interval for the Population Mean

When working in the emergency room, we took several samples of 100 patients and noticed that on average 40 of patients were treated for non-critical injuries or illnesses. The population standard deviation was 8 with an approximately normal distribution. Find a 99% confidence interval for the mean number of patients who were treated for non-critical injuries or illnesses.

8.3-8.5 Hypothesis Testing

Researchers are interested in answering many types of questions. For example,

- Is the earth warming up?
- Does a new medication lower blood pressure?
- Does the public prefer a certain color in a new fashion line?
- Is a new teaching technique better than a traditional one?
- Do seat belts reduce the severity of injuries?

These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population.

Hypothesis Testing

Three methods used to test hypotheses:

1. The traditional method
2. The *P*-value method
3. The confidence interval method

Steps in Hypothesis Testing-Traditional Method

- A **statistical hypothesis** is a conjecture about a population parameter. This conjecture may or may not be true.
- The **null hypothesis**, symbolized by H_0, is a statistical hypothesis that states that there is <u>no difference</u> between a parameter and a specific value, or that there is <u>no difference</u> between two parameters.

Steps in Hypothesis Testing-Traditional Method

- The **alternative hypothesis**, symbolized by H_1 or H_a, is a statistical hypothesis that states the existence of a <u>difference</u> between a parameter and a specific value, or states that there is a <u>difference</u> between two parameters.

Situation A

A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate increase, decrease, or remain unchanged after a patient takes the medication? The researcher knows that the mean pulse rate for the population under study is 82 beats per minute.

The hypotheses for this situation are

$$\boldsymbol{H}_0 : \mu = 82 \qquad \boldsymbol{H}_1 : \mu \neq 82$$

This is called a **two-tailed** hypothesis test.

Situation B

A chemist invents an additive to increase the life of an automobile battery. The mean lifetime of the automobile battery without the additive is 36 months.

In this book, the null hypothesis is always stated using the equals sign. The hypotheses for this situation are

$$\boldsymbol{H}_0: \mu = 36 \qquad \boldsymbol{H}_1: \mu > 36$$

This is called a **right-tailed** hypothesis test.

Situation C

A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is $78, her hypotheses about heating costs with the use of insulation are

The hypotheses for this situation are

$$H_0:\ \mu = 78 \qquad H_1:\ \mu < 78$$

This is called a **left-tailed** hypothesis test.

Claim

When a researcher conducts a study, he or she is generally looking for evidence to support a **claim**. Therefore, the claim should be stated as the alternative hypothesis, or **research hypothesis.**

A claim, though, can be stated as either the null hypothesis or the alternative hypothesis; however, the statistical evidence can only *support* the claim if it is the alternative hypothesis. Statistical evidence can be used to *reject* the claim if the claim is the null hypothesis.

These facts are important when you are stating the conclusion of a statistical study.

Hypothesis Testing

- After stating the hypotheses, the researcher's next step is to design the study. The researcher selects the **correct statistical test**, chooses an appropriate **level of significance**, and formulates a plan for conducting the study.

Hypothesis Testing

- A **statistical test** uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.
- The numerical value obtained from a statistical test is called the **test value**.
- In the hypothesis-testing situation, there are four possible outcomes.

Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.
- A **type I error** occurs if one rejects the null hypothesis when it is true.
- A **type II error** occurs if one does not reject the null hypothesis when it is false.

Hypothesis Testing

	H_0 true	H_0 false
Reject H_0	**Error** Type I	Correct decision
Do not reject H_0	Correct decision	**Error** Type II

Hypothesis Testing

- The **level of significance** is the maximum probability of committing a type I error. This probability is symbolized by α (alpha). That is,

  P(type I error) = α.

  Likewise,

  P(type II error) = β (beta).

Hypothesis Testing

- Typical significance levels are:

  0.10, 0.05, and 0.01

- For example, when $\alpha = 0.10$, there is a 10% chance of rejecting a true null hypothesis.

Hypothesis Testing

- The **critical value, C.V.**, separates the critical region from the noncritical region.
- The **critical** or **rejection region** is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.
- The **noncritical** or **nonrejection region** is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

How to Find z values for Confidence Intervals or Hypothesis Tests

Test	**Look up in Body of Table**	**Example**
Confidence interval or 2 tailed hypothesis test	α /2 (make sure to make the z value positive)	If α =.01, look up .005 in body of table and make it positive. This would give a *z* value of 2.58.
Left tailed test	α (this value should be negative)	If α =.01, look up .01 in body of table. This would give a *z* value of -2.33.
Right tailed test	α and make that value positive	α =.01, look up .01 in body of table and make it positive. This would give a *z* value of 2.33.

Hypothesis Testing

Finding the Critical Value for $\alpha = 0.01$ (Left-Tailed Test)

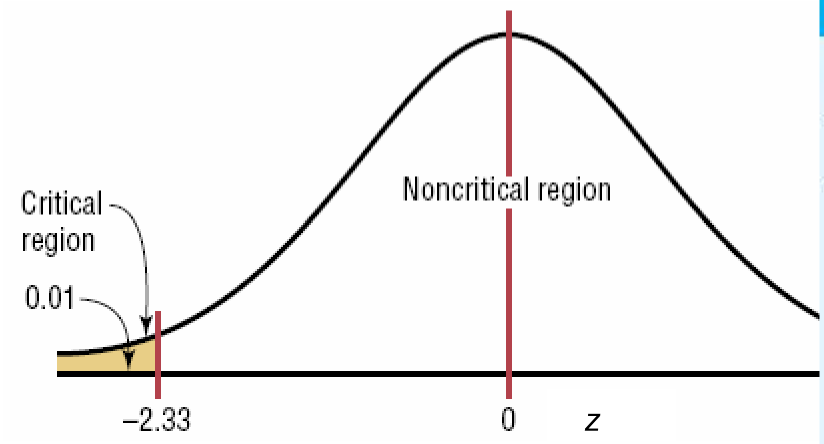


Table E The Standard Normal Distribution

Cumulative Standard Normal Distribution

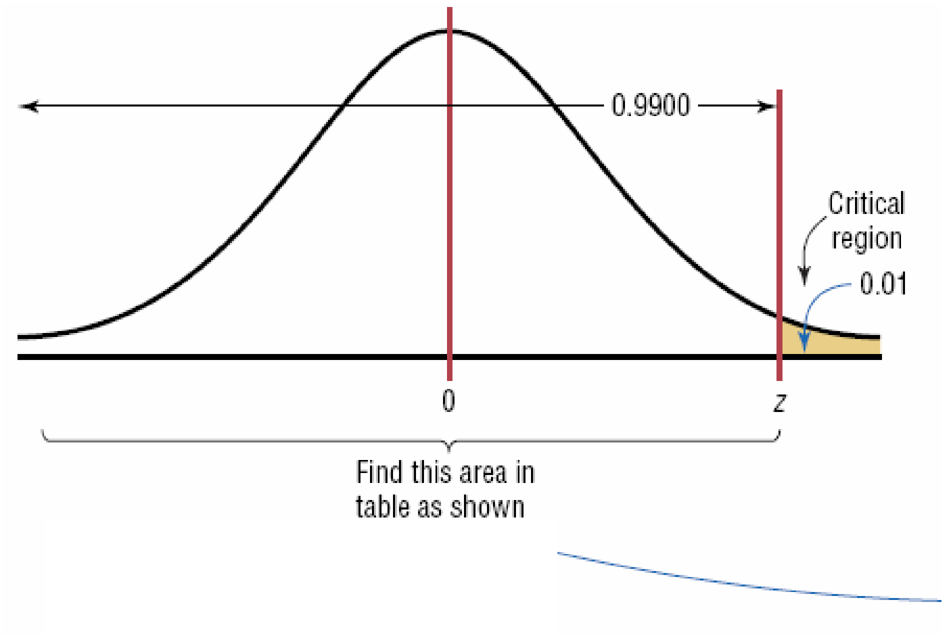
z	.00	.01	.02	.03
−3.4	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004
−3.2	.0007	.0007	.0006	.0006
−3.1	.0010	.0009	.0009	.0009
−3.0	.0013	.0013	.0013	.0012
−2.9	.0019	.0018	.0018	.0017
−2.8	.0026	.0025	.0024	.0023
−2.7	.0035	.0034	.0033	.0032
−2.6	.0047	.0045	.0044	.0043
−2.5	.0062	.0060	.0059	.0057
−2.4	.0082	.0080	.0078	.0075
−2.3	.0107	.0104	.0102	.0099

Look up the closest value to 0.01 in the middle of the table:

$z = -2.33$ for $\alpha = 0.01$ (Left-Tailed Test)

Hypothesis Testing

Finding the Critical Value for $\alpha = 0.01$ (Right-Tailed Test)

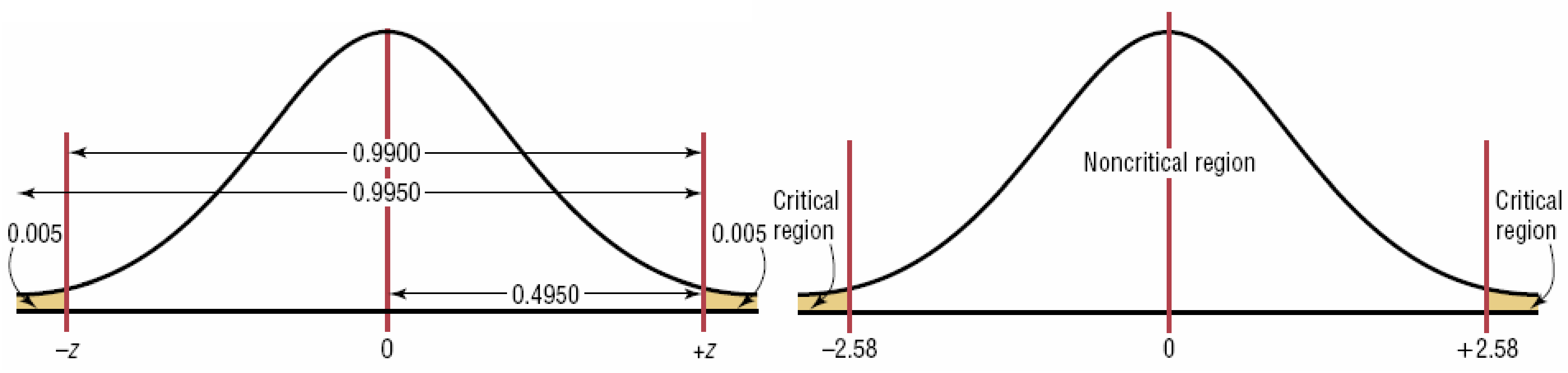


Look up the closest value to 0.01 in the table and make that z value positive:

$z = 2.33$ for $\alpha = 0.01$ (Right-Tailed Test)

Hypothesis Testing

Finding the Critical Value for $\alpha = 0.01$ (Two-Tailed Test)



Look up the closest value to 0.01/2 = 0.005 in the table and make that z value positive:

$z = 2.58$ for $\alpha = 0.01$ (Two-Tailed Test)

Procedure Table

Finding the Critical Values for Specific α Values, Using Table 3

Step 1 Draw the figure and indicate the appropriate area.

a. If the test is left-tailed, the critical region, with an area equal to α, will be on the left side of the mean.

b. If the test is right-tailed, the critical region, with an area equal to α, will be on the right side of the mean.

c. If the test is two-tailed, α must be divided by 2; one-half of the area will be to the right of the mean, and one-half will be to the left of the mean.

Procedure Table

Finding the Critical Values for Specific α Values, Using Table 3

Step 2 Find the *z* value in Table 3.

a. For a left-tailed test, use the *z* value that corresponds to the area equivalent to α in Table 3.

b. For a right-tailed test, use the *z* value that corresponds to the area equivalent to $1 - \alpha$.

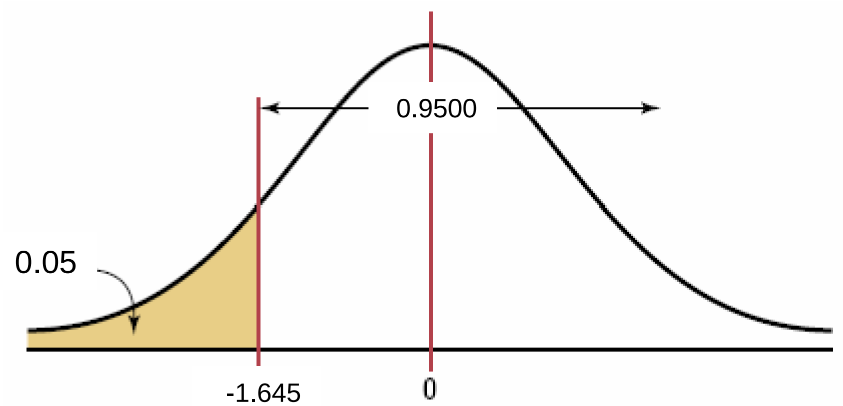
c. For a two-tailed test, use the *z* value that corresponds to $\alpha/2$ for the left value. It will be negative. For the right value, use the *z* value that corresponds to the area equivalent to $1 - \alpha/2$. It will be positive.

Ex.) 5 – Using the z table to find critical values

Using Table 3 in Appendix B, find the critical value(s) for each situation and draw the appropriate figure, showing the critical region.

a. A left-tailed test with $\alpha = 0.05$.

Step 1 Draw the figure and indicate the appropriate area.



Step 2 Find the area closest to 0.0500 in Table 3. In this case, there are 2 values:0.0495 and 0. 0505. The z values of -1.65 and -1.64 must be averaged to get -1.645.

Ex.) 5 – Using the z table to find critical values

Using Table 3 in Appendix B, find the critical value(s) for each situation and draw the appropriate figure, showing the critical region.

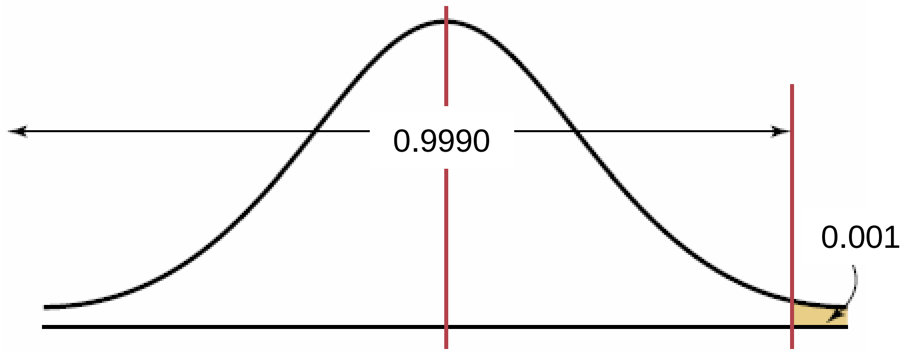
b. A two-tailed test with $\alpha = 0.05$.

Ex.) 5 – Using the z table to find critical values

Using Table 3 in Appendix B, find the critical value(s) for each situation and draw the appropriate figure, showing the critical region.

c. A right-tailed test with $\alpha = 0.001$.

Step 1 Draw the figure and indicate the appropriate area.



Step 2 Find the area closest to $\alpha = 0.001$.
The closest value is -3.09. Making that positive gives 3.09.

Common Critical Values for Tests

Test	α	***z***	***t***
2 tailed	0.01	2.58	Look up based on degrees of freedom, d.f. = $n - 1$ for most tests.
	0.05	1.96	
	0.10	1.64	
Left tailed	0.01	-2.33	
	0.05	-1.64	
	0.10	-1.28	
Right tailed	0.01	2.33	
	0.05	1.64	
	0.10	1.28	

Note: we will discuss the t value in Ch 9

Procedure Table

Solving Hypothesis-Testing Problems
(Traditional Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table in Appendix B.

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

z Test for a Mean μ (σ Known)

The z test is a statistical test for the mean of a population. It can be used when $n \geq 30$, or when the population is normally distributed and σ is known.

The formula for the *z test is*

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\bar{X}$ = sample mean

μ = hypothesized population mean

σ = population standard deviation

n = sample size

Ex.) 6 – z Test for a Mean

Peanut Production in Virginia The average production of peanuts in Virginia is 3000 pounds per acre. A new plant food has been developed and is tested on 60 individual plots of land. The mean yield with the new plant food is 3120 pounds of peanuts per acre, and the population standard deviation is 578 pounds. At $\alpha = 0.05$, can you conclude that the average production has increased?

Step 1: State the hypotheses and identify the claim.

H_0: μ = 3,000 lbs/acre *and* H_1: μ > 3,000 lbs/acre (claim)

Step 2: Find the critical value.

Since α = 0.05 and the test is a right-tailed test, the critical value is z = 1.65.

Ex.) 6 – z Test for a Mean

Peanut Production in Virginia The average production of peanuts in Virginia is 3000 pounds per acre. A new plant food has been developed and is tested on 60 individual plots of land. The mean yield with the new plant food is 3120 pounds of peanuts per acre, and the population standard deviation is 578 pounds. At $\alpha = 0.05$, can you conclude that the average production has increased?

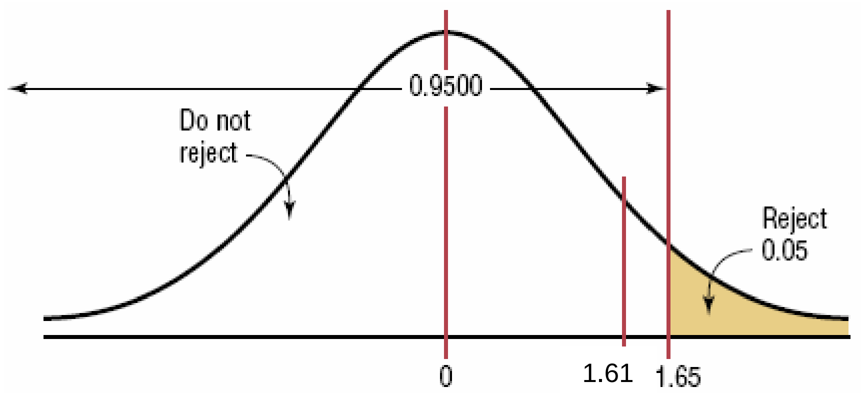
Step 3: Compute the test value.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{3120 - 3000}{578 / \sqrt{60}} = 1.61$$

Ex.) 6 – z Test for a Mean

Step 4: Make the decision.

Since the test value, 1.61, is less than the critical value, 1.65, and is not in the critical region, the decision is to not reject the null hypothesis.



Step 5: Summarize the results.

There is not enough evidence to support the claim that the average production has increased.

Important Comments

- Even though in Example 6 the sample mean of 3120 lbs is higher than the hypothesized population mean of 3000 lbs, it is not significantly higher. Hence, the difference may be due to chance.
- When the null hypothesis is not rejected, there is still a probability of a type II error, i.e., of not rejecting the null hypothesis when it is false.
- When the null hypothesis is not rejected, it cannot be accepted as true. There is merely not enough evidence to say that it is false.

Ex.) 7 – z Test for a Mean

Salaries of Government Employees The mean salary of federal government employees on the General Schedule is \$59,593. The average salary of 30 state employees who do similar work is \$58,800 with $\sigma = \$1500$. At the 0.01 level of significance, can it be concluded that state employees earn on average less than federal employees?

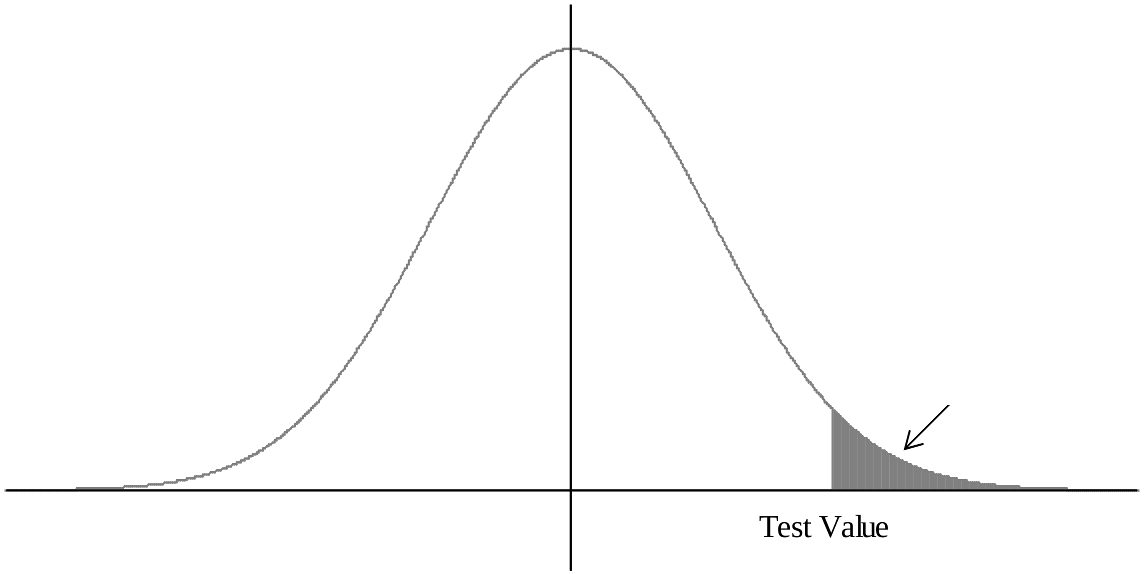
Ex.) 8 – z Test for a Mean

Heights of 1-Year-Olds The average 1-year-old (both genders) is 29 inches tall. A random sample of 30 one-year-olds in a large day care franchise resulted in the following heights. At $\alpha = 0.05$, can it be concluded that the average height differs from 29 inches? Assume $\sigma = 2.61$.

25	32	35	25	30	26.5	26	25.5	29.5	32
30	28.5	30	32	28	31.5	29	29.5	30	34
29	32	27	28	33	28	27	32	29	29.5

Hypothesis Testing

The ***P*-value** (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.



Hypothesis Testing

- In this section, the traditional method for solving hypothesis-testing problems compares ***z*-values**:
  - critical value
  - test value
- The *P*-value method for solving hypothesis-testing problems compares **areas**:
  - alpha
  - *P*-value

Procedure Table

Solving Hypothesis-Testing Problems
(*P*-Value Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the *P*-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Ex.) 9 – P-value Method

Repeat Ex.) 6 using the P-value method.

Step 1: State the hypotheses and identify the claim.

H_0: μ = 3,000 lbs/acre *and* H_1: μ > 3,000 lbs/acre (claim)

Step 2: Compute the test value.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{3120 - 3000}{578 / \sqrt{60}} = 1.61$$

Step 3: Find the *P*-value.

Using Table 3, find the area for z = 1.61.

The area is 0.9463.

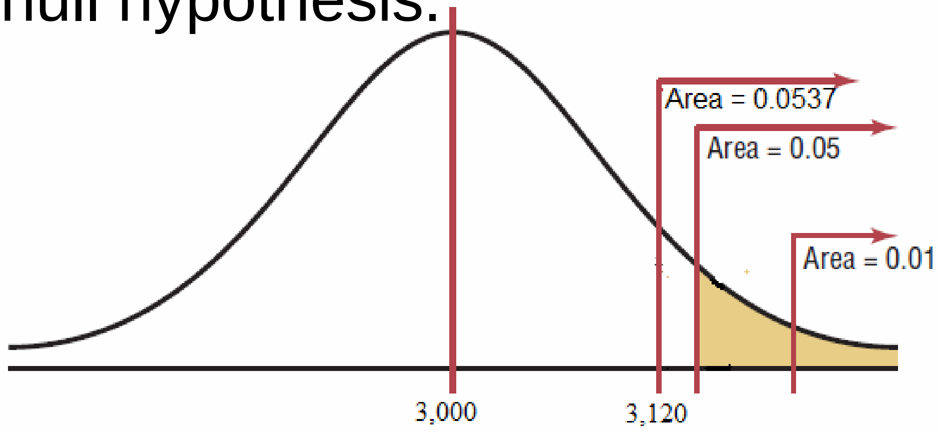
Subtract from 1.0000 to find the area of the tail.

Hence, the *P*-value is 1.0000 – 0.9463 = 0.0537.

Ex.) 9 – P-value Method

Step 4: Make the decision.

Since the *P*-value is greater than 0.05, the decision is to accept the null hypothesis.



Step 5: Summarize the results.

There is not enough evidence to support the claim that the average production has increased.

Note: If $\alpha = 0.10$, the null hypothesis would be rejected.

Ex.) 10 – P-value Method

Repeat Ex.) 8 using the P-value method.

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 29$ *and* H_1: $\mu \neq 29$ (claim)

Step 2: Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{29.45 - 29}{2.61 / \sqrt{30}} = 0.944$$

Step 3: Find the *P*-value.

The area for $z = 0.944$ is 0.8264.

Subtract: 1.0000 – 0.8264 = 0.1736.

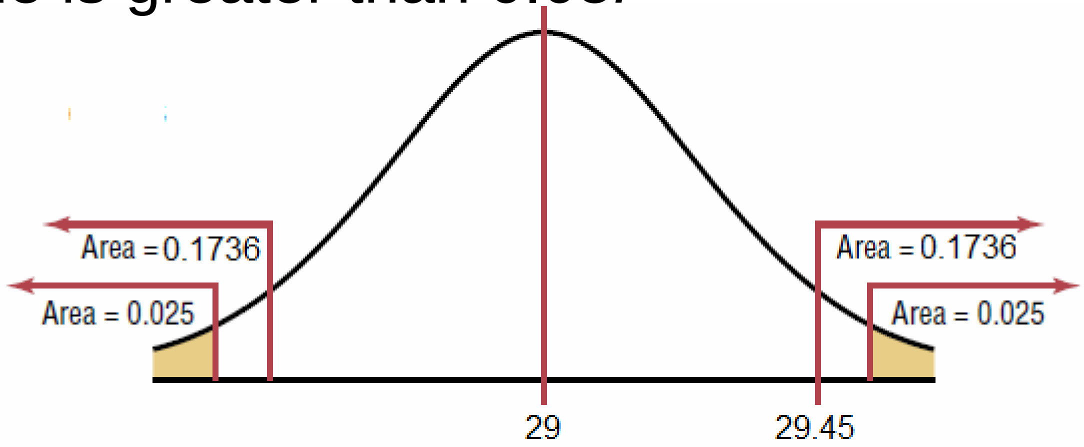
Since this is a two-tailed test, the area of 0.1736 must be doubled to get the *P*-value.

The *P*-value is 2(0.1736) = 0.3472.

Ex.) 10 – P-value Method

Step 4: Make the decision.

The decision is to not reject the null hypothesis, since the *P*-value is greater than 0.05.



Step 5: Summarize the results.

There is not enough evidence to support the claim that the average height of 1-year olds differs from 29 inches.

Guidelines for *P*-Values With No α

- If *P*-value $\leq$ 0.01, reject the null hypothesis. The difference is highly significant.

- If P-value > 0.01 but P-value $\leq$ 0.05, reject the null hypothesis. The difference is significant.

- If P-value > 0.05 but P-value $\leq$ 0.10, consider the consequences of type I error before rejecting the null hypothesis.

- If P-value > 0.10, do not reject the null hypothesis. The difference is not significant.

Significance

- The researcher should distinguish between **statistical significance** and **practical significance**.
- When the null hypothesis is rejected at a specific significance level, it can be concluded that the difference is probably not due to chance and thus is statistically significant. However, the results may not have any practical significance.
- It is up to the researcher to use common sense when interpreting the results of a statistical test.

Class Activity: Hypothesis Testing for the Population Mean

When working in the emergency room, we took samples of 100 patients and noticed that on average 40 of patients were treated for non-critical injuries or illnesses. The population standard deviation was 8 with an approximately normal distribution. When we conducted this study 5 years ago, we found the average number of patients treated for non-critical injuries or illnesses to be 45. At a level of $\alpha = 0.05$, can we conclude that the number of patients being treated for non-critical injuries or illnesses in the emergency room has decreased?