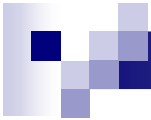




Chapter 6

The Normal Probability Distributions



Chapter 6 Overview

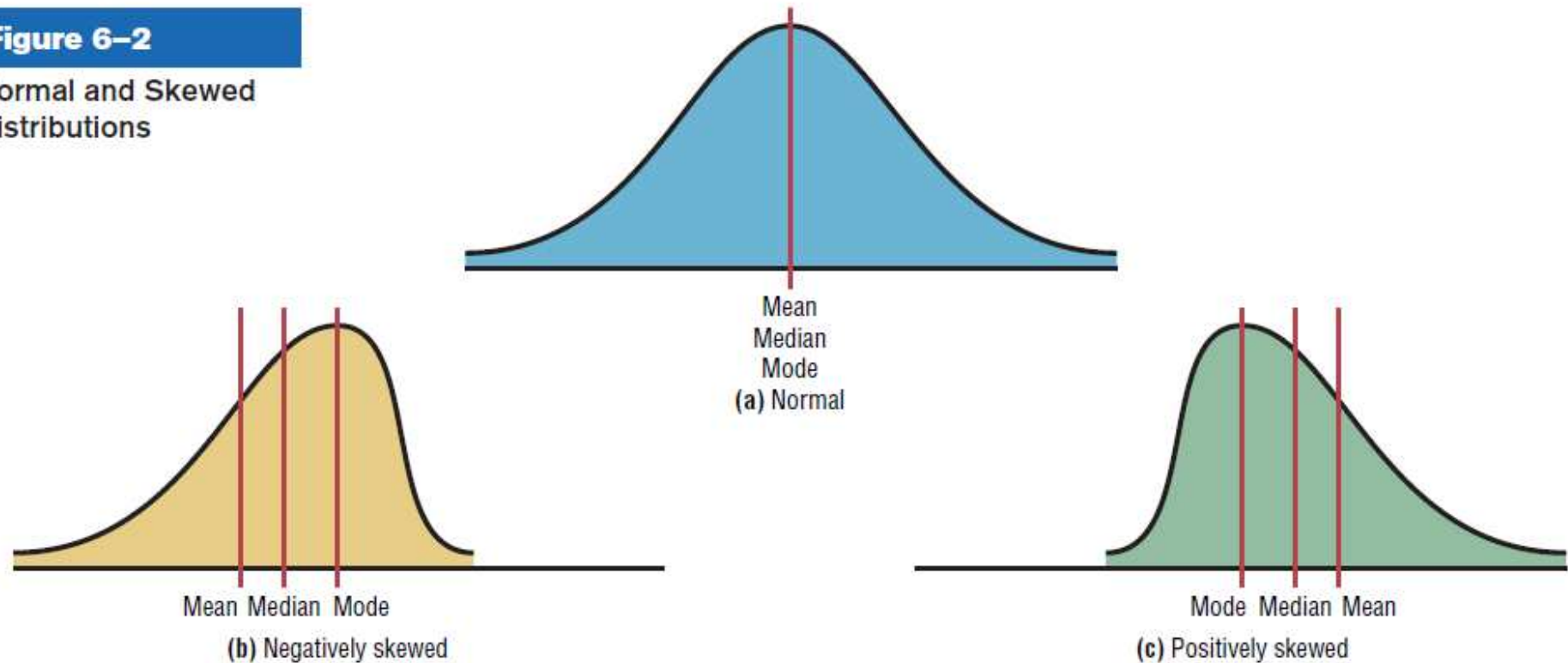
Introduction

- 6-1 Normal Probability Distributions
- 6-2 The Standard Normal Distribution
- 6-3 Applications of the Normal Distribution
- 6-5 The Normal Approximation to the Binomial Distribution

Types of Distributions

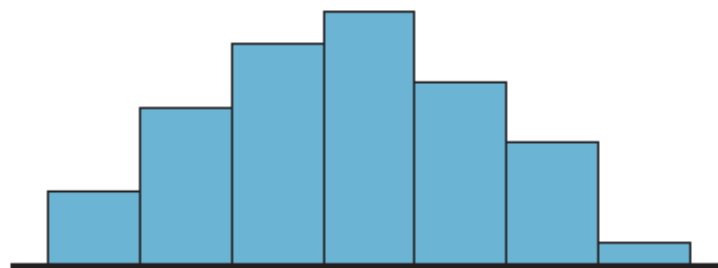
Figure 6-2

Normal and Skewed
Distributions

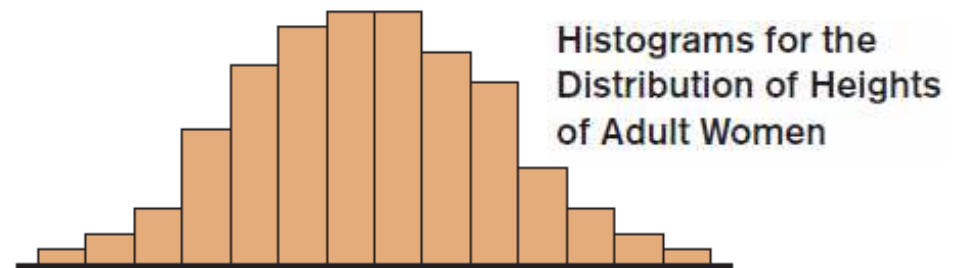


6.1 Normal Distributions

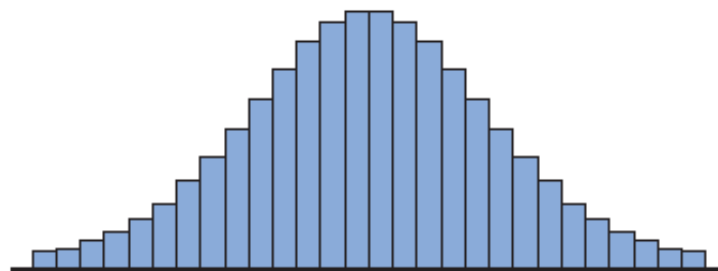
- Many continuous variables have distributions that are bell-shaped and are called **approximately normally distributed variables**.



(a) Random sample of 100 women



(b) Sample size increased and class width decreased



(c) Sample size increased and class width decreased further



(d) Normal distribution for the population

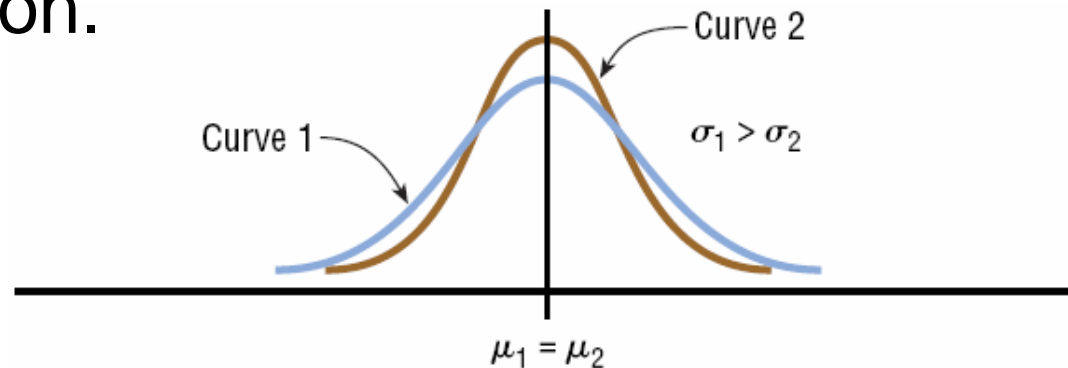


6-1 Normal Distributions

- The theoretical curve, called the **bell curve** or the **Gaussian distribution**, can be used to study many variables that are not normally distributed but are approximately normal.

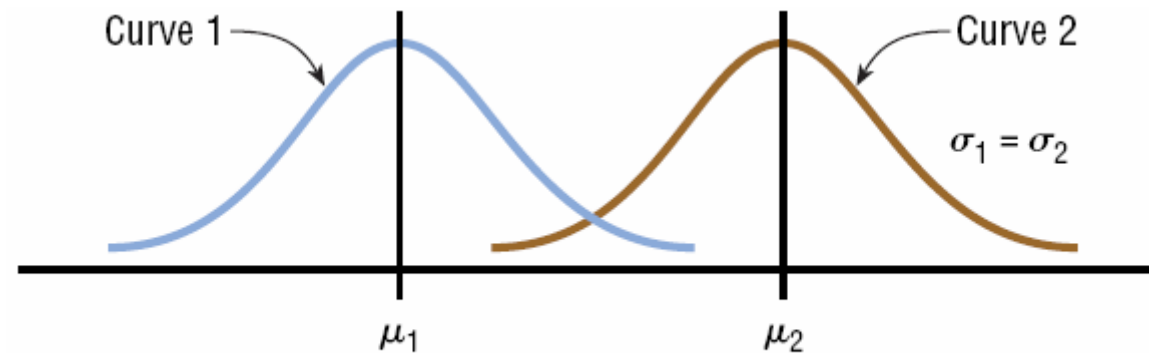
Normal Distributions

- The shape and position of the normal distribution curve depend on two parameters, the **mean** and the **standard deviation**.
- Each normally distributed variable has its own normal distribution curve, which depends on the values of the variable's mean and standard deviation.

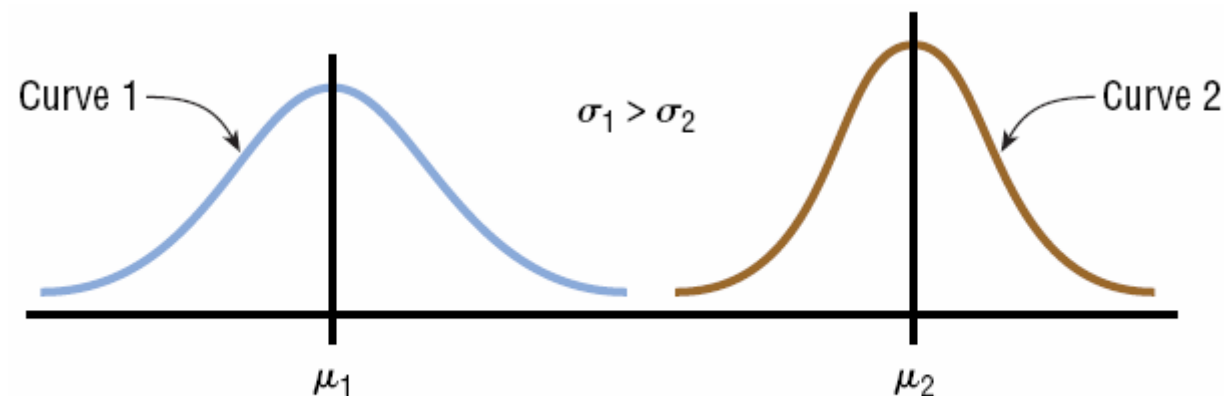


(a) Same means but different standard deviations

Normal Distributions



(b) Different means but same standard deviations



(c) Different means and different standard deviations



Normal Distribution Properties

- The normal distribution curve is bell-shaped.
- The mean, median, and mode are equal and located at the center of the distribution.
- The normal distribution curve is **unimodal** (i.e., it has only one mode).
- The curve is symmetrical about the mean, which is equivalent to saying that its shape is the same on both sides of a vertical line passing through the center.



Normal Distribution Properties

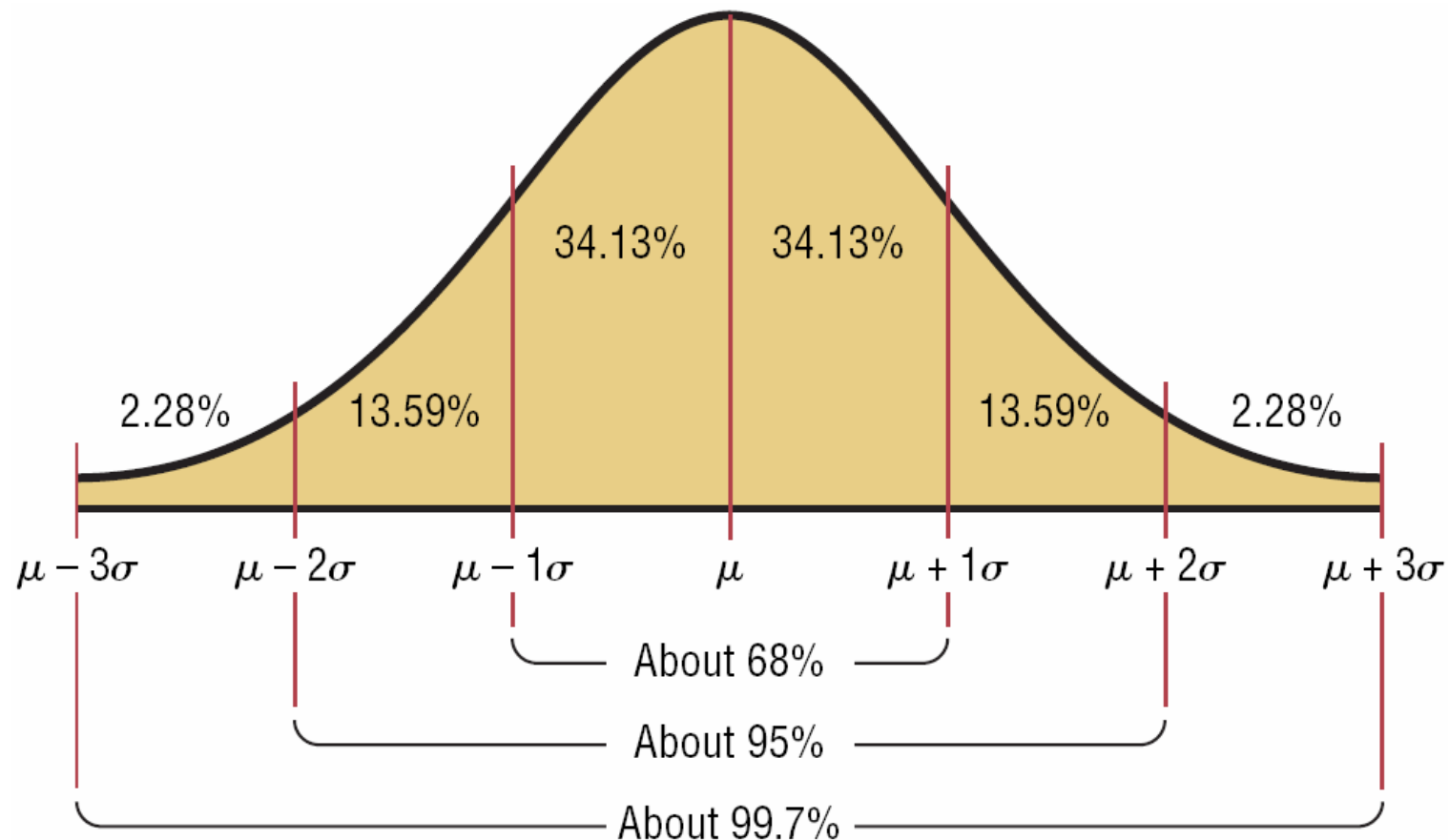
- The curve is continuous—i.e., there are no gaps or holes. For each value of X , there is a corresponding value of Y .
- The curve never touches the x axis.
Theoretically, no matter how far in either direction the curve extends, it never meets the x axis—but it gets increasingly closer.



Normal Distribution Properties

- The total area under the normal distribution curve is equal to 1.00 or 100%.
- The area under the normal curve that lies within
 - one standard deviation of the mean is approximately 0.68 (68%).
 - two standard deviations of the mean is approximately 0.95 (95%).
 - three standard deviations of the mean is approximately 0.997 (99.7%).

Normal Distribution Properties



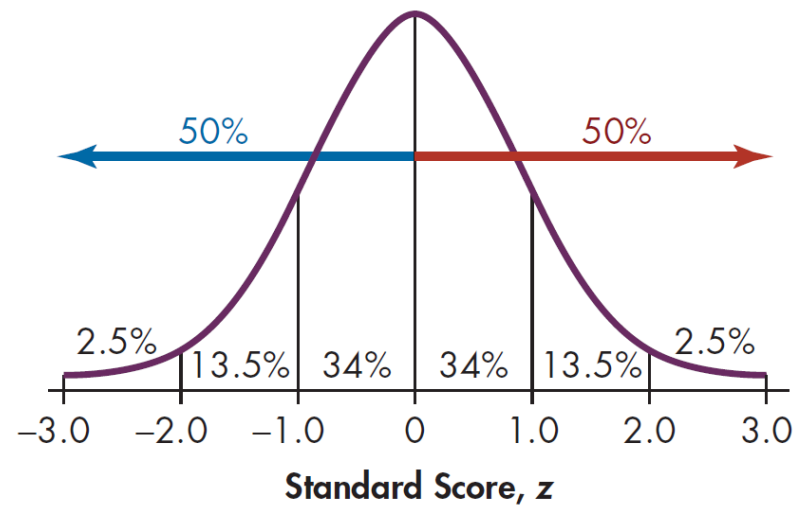


6-2 Standard Normal Distribution

- Since each normally distributed variable has its own mean and standard deviation, the shape and location of these curves will vary. In practical applications, one would have to have a table of areas under the curve for each variable. To simplify this, statisticians use the standard normal distribution.
- The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

Standard Normal Distribution

When using the empirical rule, the values of z were typically integer values; see Figure 6.3.



Standard Normal Distribution According to Empirical Rule

Figure 6.3



z score (Standard Score)

The z score is the number of standard deviations that a particular X value is away from the mean.
The formula for finding the z score is:

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

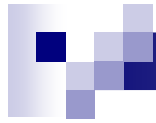
$$z = \frac{X - \mu}{\sigma}$$

Looking up a z value in the Table

- To look up the z value of 1.39 in the table, locate the 1.3 row and the 0.09 column. Then find the corresponding value.

<i>z</i>	0.00	...	0.09
0.0			
⋮			
1.3			0.9177
⋮			

- Note: This is found in Table 3 in Appendix B



Looking up z values on the table

- Look up the following z values on the standard normal distribution table:

- ☐ 2.25

- ☐ -1.56

- ☐ 0.52

- ☐ -3.62



Class Activity

- Look up the following z values on the standard normal distribution table:

- ☐ 1.89

- ☐ -2.75

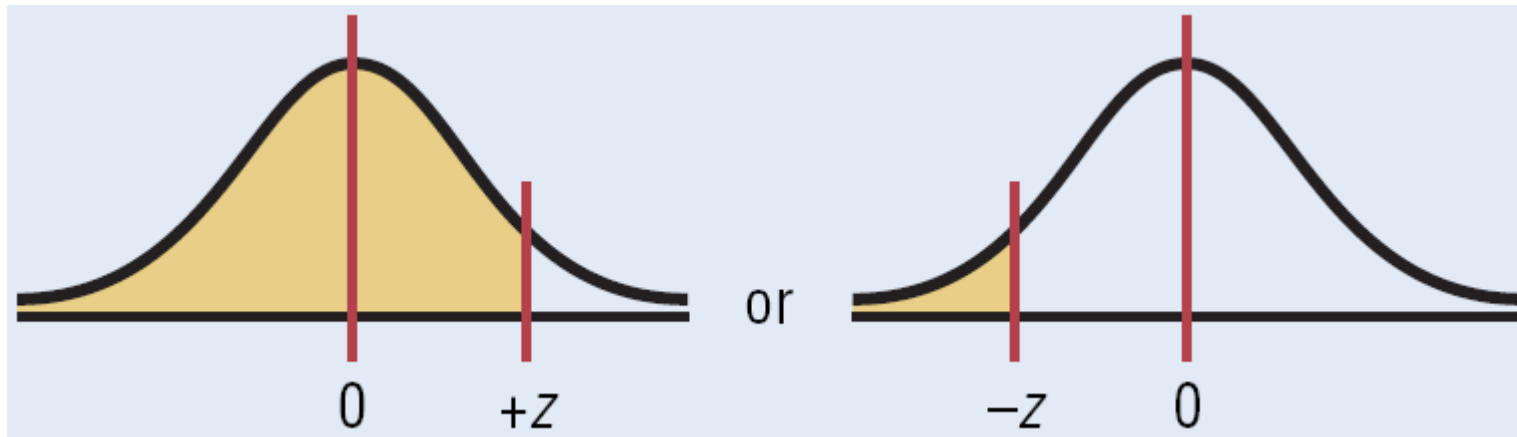
- ☐ -0.55

- ☐ 2.63

Area under the Standard Normal Distribution Curve

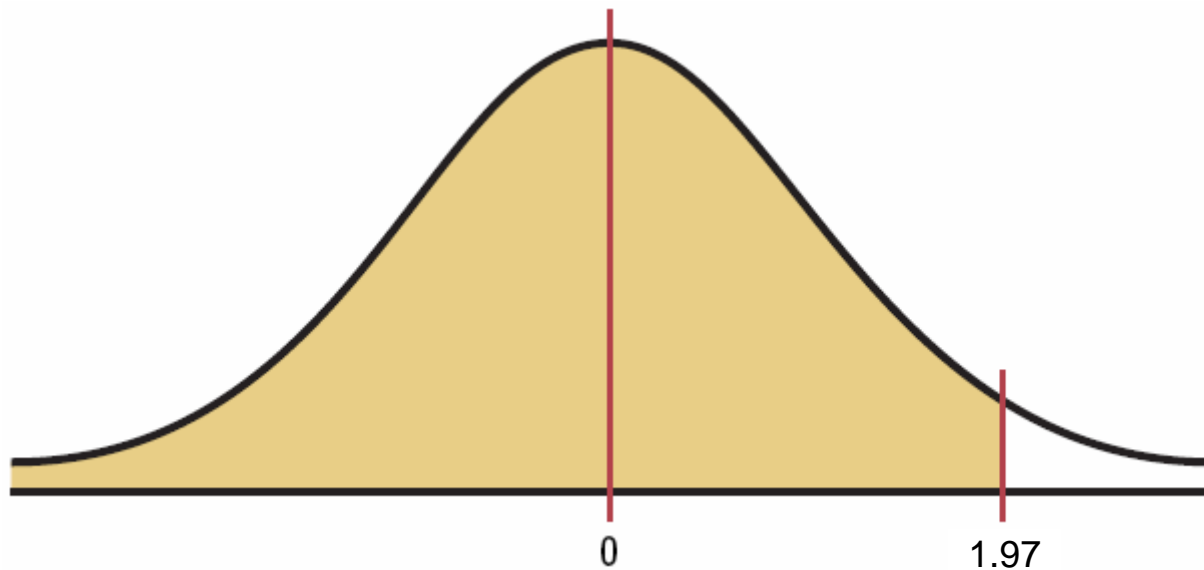
1. To the left of any z value:

Look up the z value in the table and use the area given.



Ex.) 1 – Area under the Curve

Find the area to the left of $z = 1.97$.



The value in the 1.9 row and the .07 column of Table 3 is .9756. The area is .9756.



Area under the Standard Normal Distribution Curve

- Find the area to the left of $z = -0.46$.



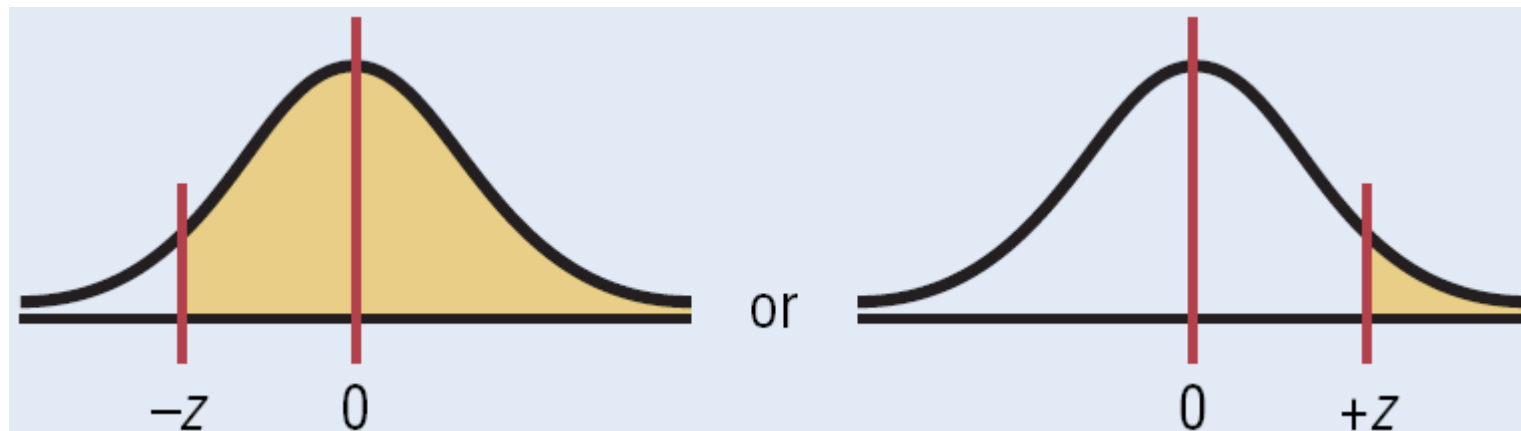
Area under the Standard Normal Distribution Curve

- Find the area to the left of $z = 2.00$.

Area under the Standard Normal Distribution Curve

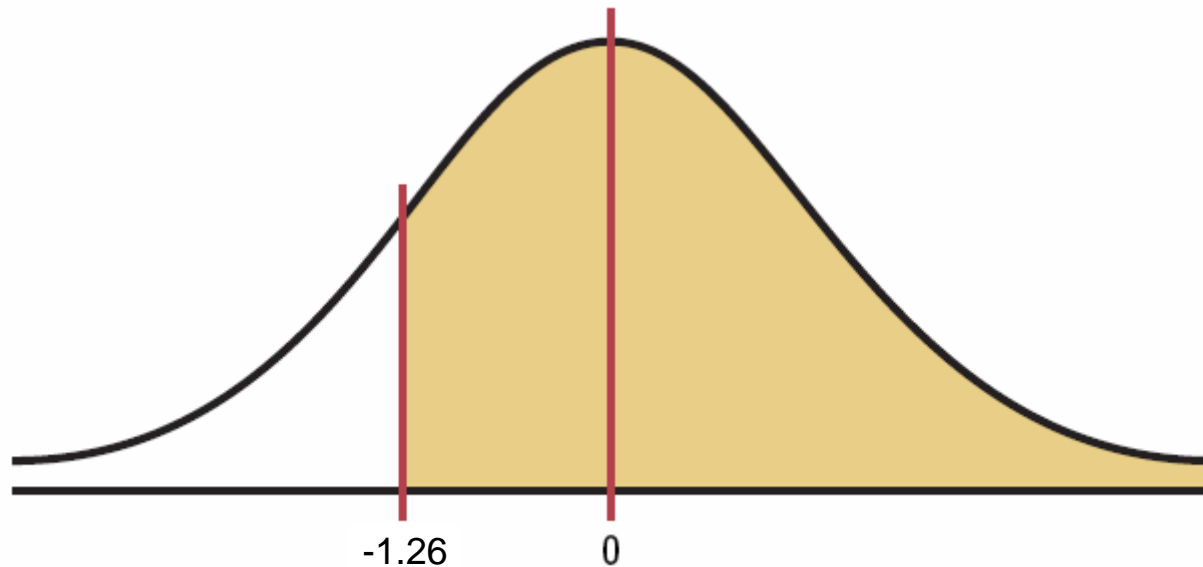
2. To the right of any z value:

Look up the negative of the z value.



Ex.) 2 – Area under the Curve

Find the area to right of $z = -1.26$.



Lookup up the negative of -1.26 , which is 1.26 ,
The value in the 1.2 row and the $.06$ column of
Table 3 is $.8962$.



Area under the Standard Normal Distribution Curve

- Find the area to the right of $z = 2.40$.



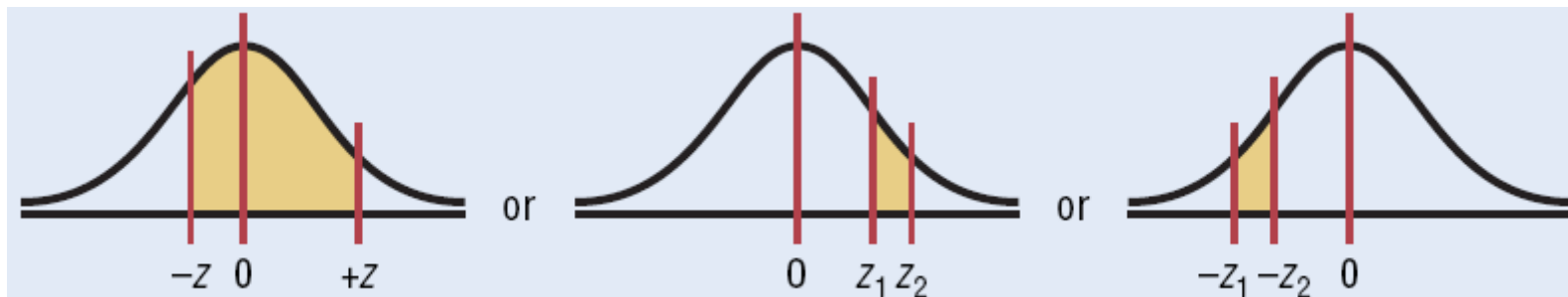
Area under the Standard Normal Distribution Curve

- Find the area to the right of $z = -1.86$.

Area under the Standard Normal Distribution Curve

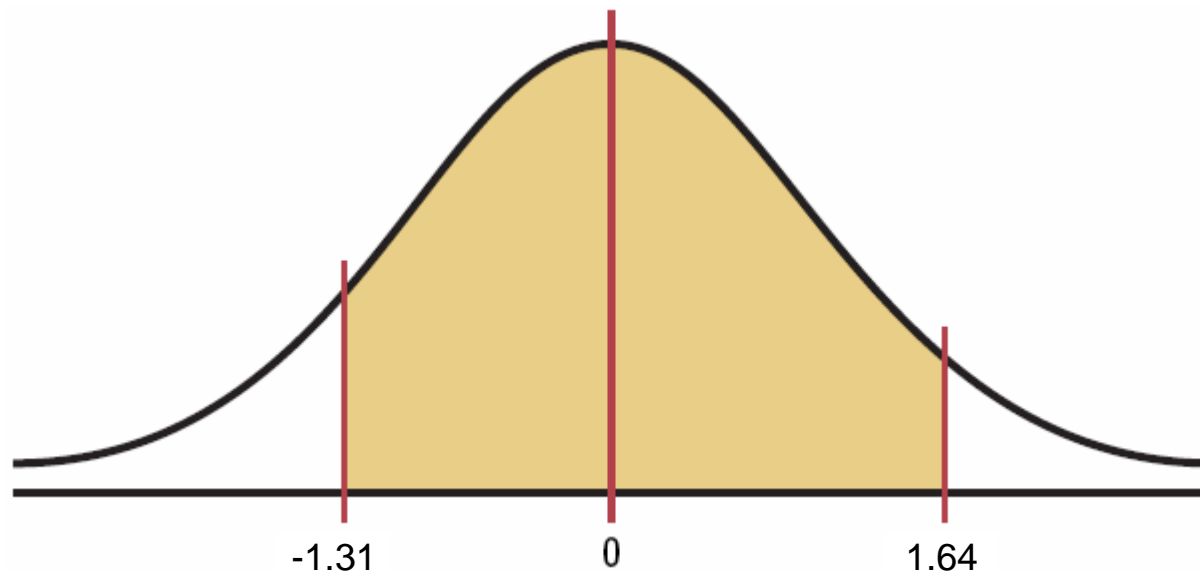
3. Between two z values:

Look up both z values and subtract the corresponding areas.



Ex.) 3 – Area under the Curve

Find the area between $z = 1.64$ and $z = -1.31$.



The values for $z = 1.64$ is .9495 and for $z = -1.31$ is .0951. The area is $.9495 - .0951 = .8544$.



Area under the Standard Normal Distribution Curve

- Find the area between $z = 1.23$ and $z = 1.90$

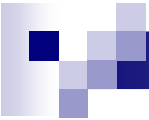
.



Area under the Standard Normal Distribution Curve

- Find the area between $z = -1.45$ and $z = 1.87$

.

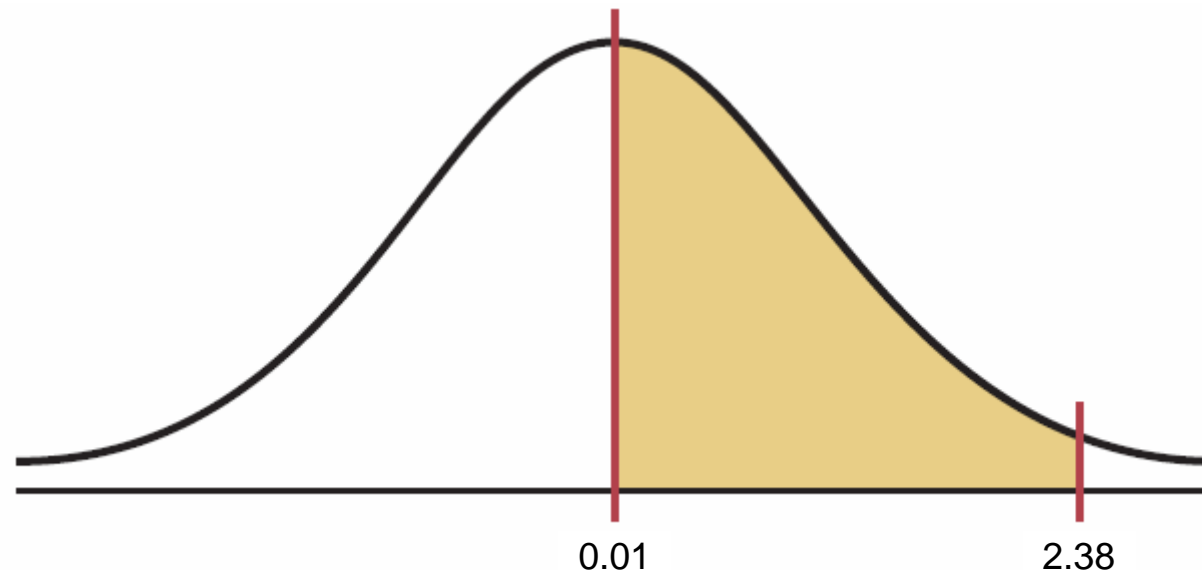


Probability under the Normal Distribution Curve

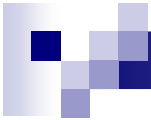
- For the Normal Distribution, probability and area are the same thing.
 - Area to the left of a z value is the same as a probability less than a z value
 - $P(z < a)$
 - Area to the right of a z value is the same as a probability greater than a z value
 - $P(z > a)$
 - Area between two z values is the same as a probability between two z values
 - $P(a < z < b)$

Ex.) 4 – Probability

Find the probability: $P(0.01 < z < 2.38)$

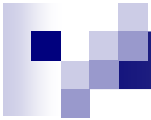


The values for $z = 2.38$ is .9913 and for $z = 0.01$ is .5040. The probability is $.9913 - .5040 = .4873$.




Probability under the Normal Distribution Curve

- Find $P(-2.05 < z < 1.85)$




Probability under the Normal Distribution Curve

- Find $P(z > -0.5)$



Probability under the Normal Distribution Curve

- Find $P(-1.25 < z < 0)$

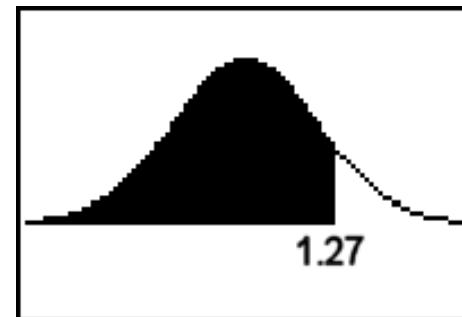
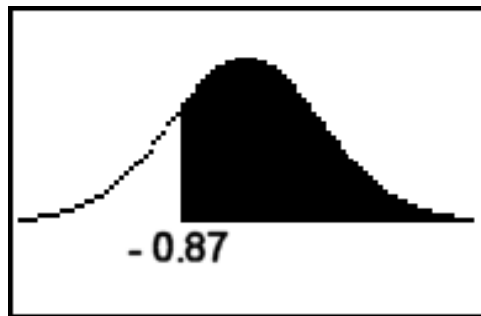
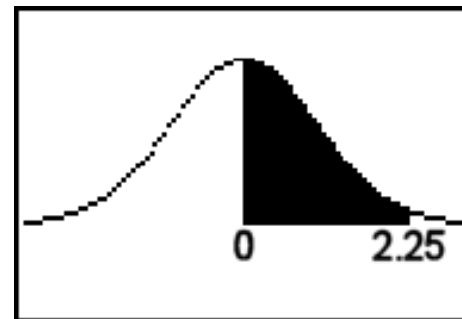
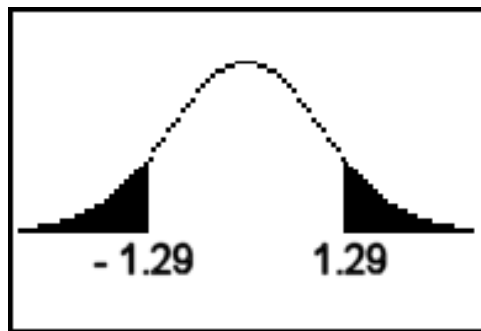



Probability under the Normal Distribution Curve

- Find $P(z < -1.54 \text{ or } z > 1.54)$

Probability under the Normal Distribution Curve

- Find the probabilities of the shaded regions.



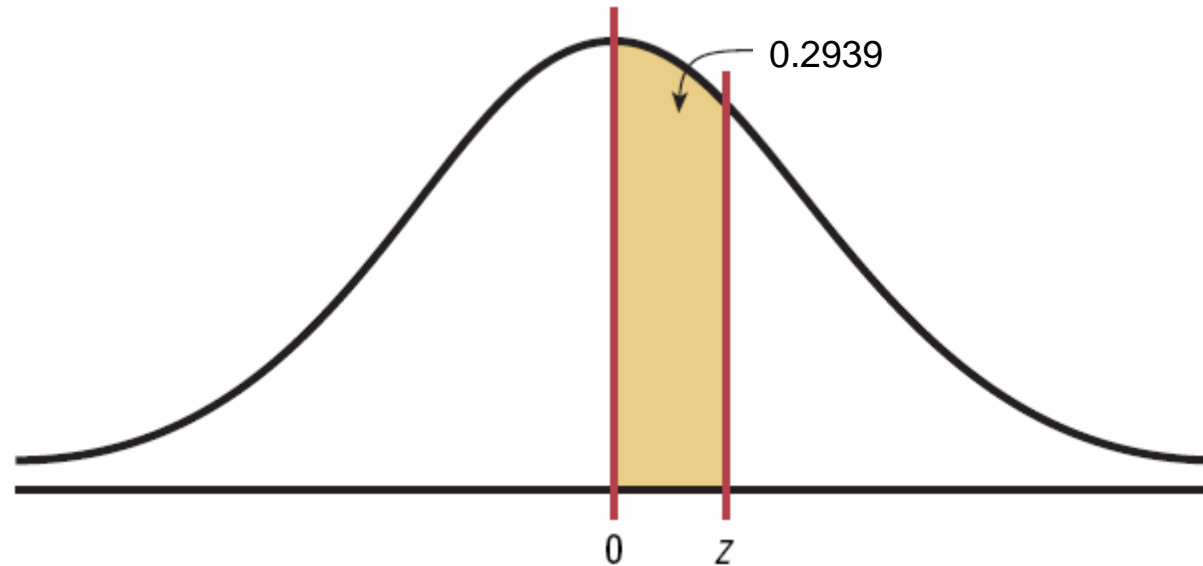


Probability under the Normal Distribution Curve

- We can also find a z value if we are given an area or probability.
- This is called the reverse lookup problem.
- What we have been doing up until now is called the forward lookup problem.

Ex.) 5 – Probability

Find the z value such that the area under the standard normal distribution curve between 0 and the z value is 0.2939.



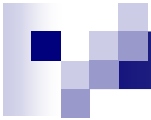
Add .5000 to .2939 to get the cumulative area of .7939. Then look for that value inside Table 3.

Ex.) 5 – Probability

Add .5000 to .2939 to get the cumulative area of .7939. Then look for that value inside Table 3.

<i>z</i>	.00	.01	.02	.03
0.0	.5000	.5040	.5080	.5120
0.1	.5398	.5438	.5478	.5517
0.2	.5793	.5832	.5871	.5910
0.3	.6179	.6217	.6255	.6293
0.4	.6554	.6591	.6628	.6664
0.5	.6915	.6950	.6985	.7019
0.6	.7257	.7291	.7324	.7357
0.7	.7580	.7611	.7642	.7673
0.8	.7881	.7910	.7939	.7967
0.9	.8159	.8186	.8212	.8238
1.0	.8413	.8438	.8461	.8485

The *z* value is 0.82.

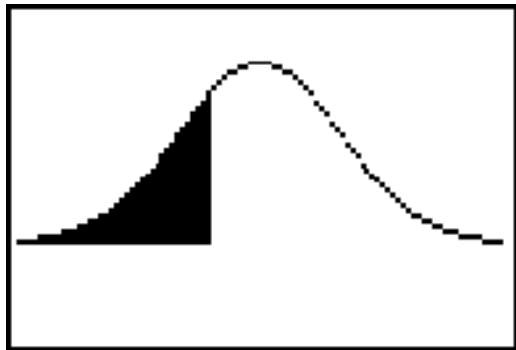


Probability under the Normal Distribution Curve

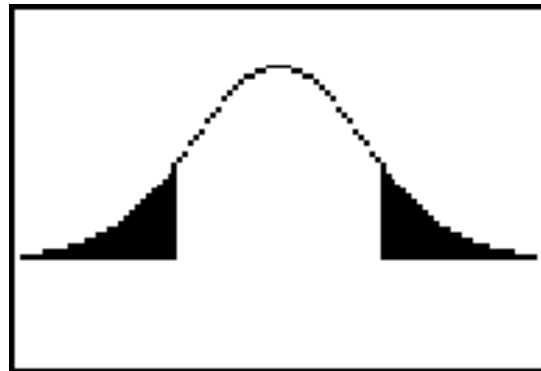
- Find the z score(s) for the corresponding area or probability:
 - ☐ the 64th percentile
 - ☐ the middle 90% of the data

Probability under the Normal Distribution Curve

- Find the indicated z-score(s) indicated by the graph.



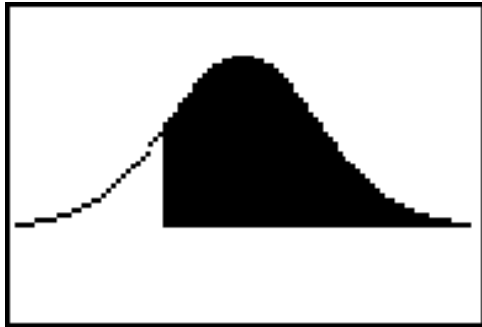
Area = 0.2776



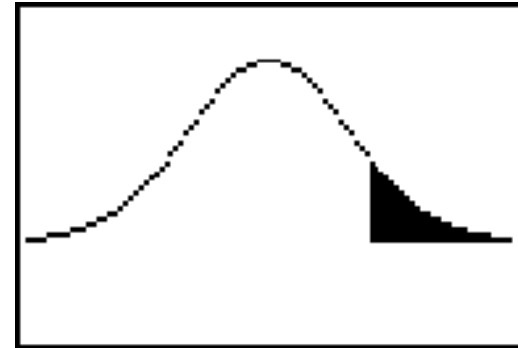
Area on each end = 0.1251

Probability under the Normal Distribution Curve

- Find the indicated z-score(s) indicated by the graph.



Area = 0.8508



Area = .1093



Class Activity: Normal Distribution Table

Find the area under the standard normal distribution curve:

Between $z = -1.53$ and $z = -2.08$

To the right of $z = 1.92$ and to the left of $z = -0.44$

Find the probabilities:

$P(-2.46 < z < 1.74)$

$P(z < 1.42)$

Find the following:

Find two z values so that 48% of the middle area is bounded by them.



6-3 Applications of the Normal Distribution

- The standard normal distribution curve can be used to solve a wide variety of practical problems. The only requirement is that the variable be normally or approximately normally distributed.
- For all the problems presented in this chapter, you can assume that the variable is normally or approximately normally distributed.



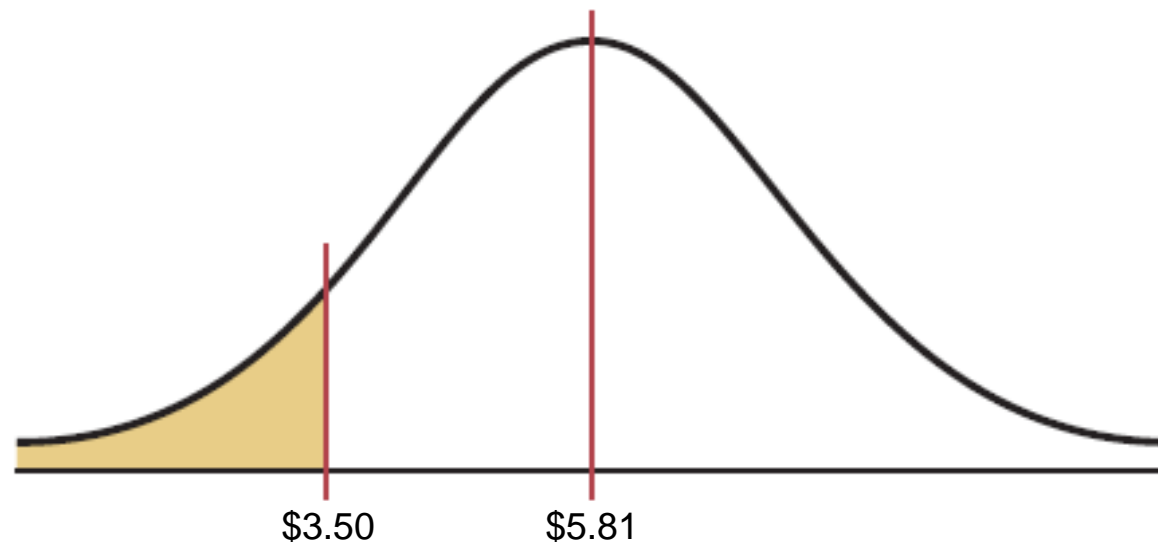
Applications of the Normal Distribution

- To solve problems by using the standard normal distribution, transform the original variable to a standard normal distribution variable by using the z value formula.
- This formula transforms the values of the variable into standard units or z values. Once the variable is transformed, then the Procedure Table and Table 3 in Appendix B can be used to solve problems.

Ex.) 6 – Applications of the Normal Distribution

Admission Charge for Movies The average admission charge for a movie is \$5.81. If the distribution of movie admission charges is approximately normal with a standard deviation of \$0.81, what is the probability that a randomly selected admission charge is less than \$3.50?

Step 1: Draw the normal distribution curve.



Ex.) 6 – Applications of the Normal Distribution

Step 2: Find the z value corresponding to \$3.50.

$$z = \frac{X - \mu}{\sigma} = \frac{3.50 - 5.81}{0.81} = -2.85$$

Step 3: Find the area to the left of $z = -2.85$.

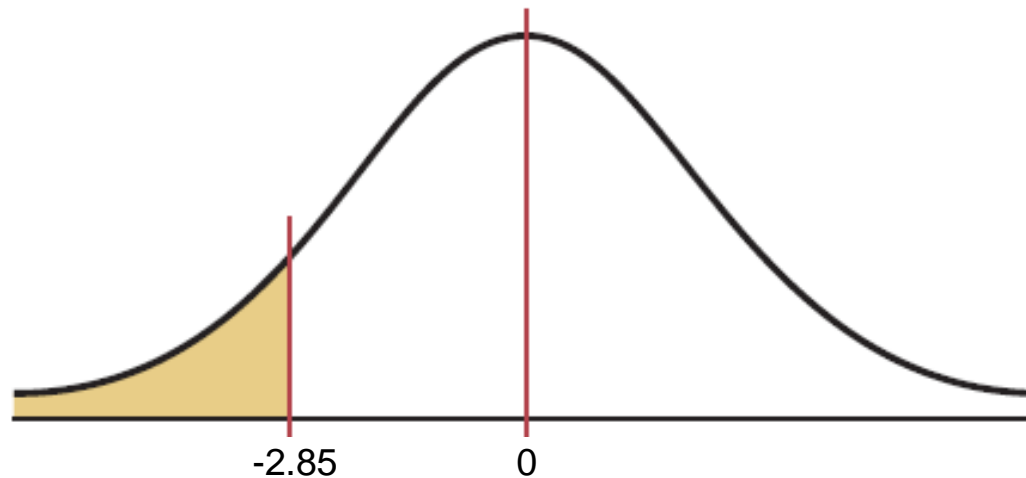


Table 3 gives us an area of 0.0022.

0.22% of movie tickets cost less than \$3.50.



Applications of the Normal Distribution

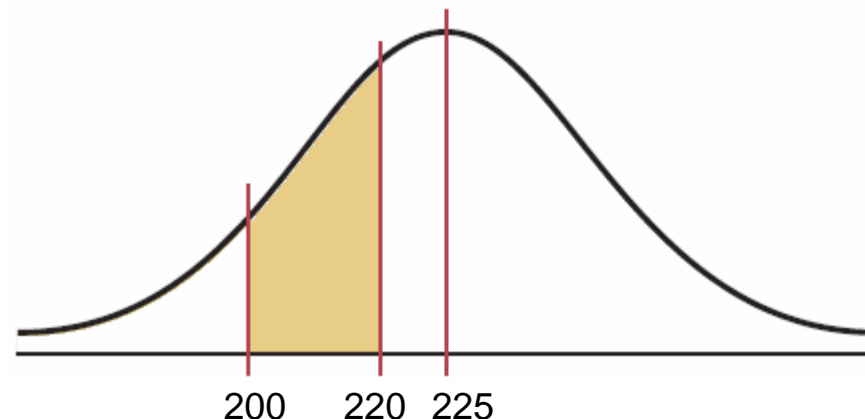
- Insurance companies have determined that US males between the ages of 16 and 24, drive an average of 10,718 miles each year with a standard deviation of 3763 miles. Assume the data is normally distributed. For a randomly selected male in that age group, find the probability that he drives less than 12,000 miles per year.

Ex.) 7 – Applications of the Normal Distribution

Chocolate Bar Calories The average number of calories in a 1.5-ounce chocolate bar is 225. Suppose that the distribution of calories is approximately normal with $\sigma = 10$. Find the probability that a randomly selected chocolate bar will have

a. Between 200 and 220 calories

Step 1: Draw the normal distribution curve.



Ex.) 7 – Applications of the Normal Distribution

Step 2: Find z values corresponding to 200 and 220.

$$z = \frac{220 - 225}{10} = -0.50 \qquad z = \frac{200 - 225}{10} = -2.50$$

Step 3: Find the area between $z = -2.5$ and $z = -0.5$.

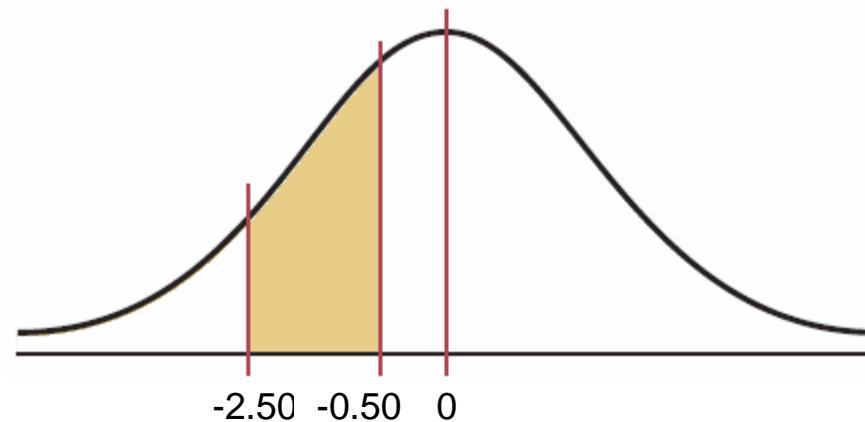


Table 3 gives us an area of $.3085 - .0062 = .3023$.

The probability is 30.23%.



Applications of the Normal Distribution

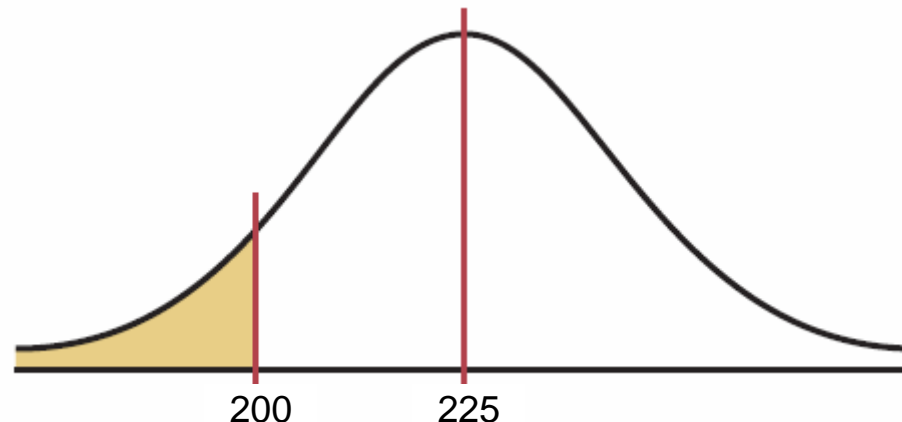
- The heights of six-year old girls are normally distributed with a mean of 117.80 cm and a standard deviation of 5.52 cm. Find the probability that a randomly selected six-year girl has a height between 117.80 cm and 120.56 cm.

Ex.) 8 – Applications of the Normal Distribution

Chocolate Bar Calories The average number of calories in a 1.5-ounce chocolate bar is 225. Suppose that the distribution of calories is approximately normal with $\sigma = 10$. Find the probability that a randomly selected chocolate bar will have Less than 200 calories

In a sample of 400 chocolate bars, how many will have less than 200 calories?

Step 1: Draw the normal distribution curve.





Ex.) 8 – Applications of the Normal Distribution

Step 2: Find the z value for 200.

$$z = \frac{200 - 225}{10} = -2.50$$

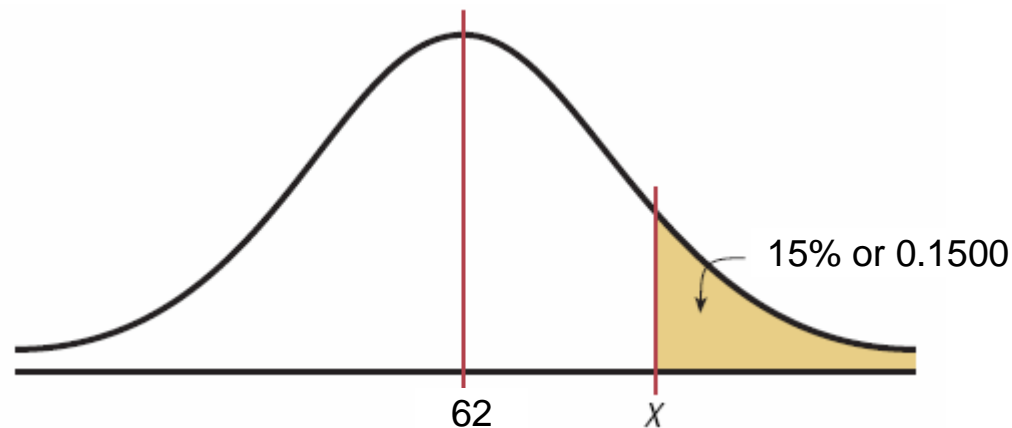
Step 3: Find the area to the left of $z = -2.50$. It is 0.0062.

Step 4: To find how many chocolate bars have less than 200 calories, multiply the sample size 400 by 0.0062 to get 2.48. Hence, approximately 2 chocolate bars will be under 200 calories.

Ex.) 9 – Applications of the Normal Distribution

Security Officer Stress Tolerance To qualify for security officers' training, recruits are tested for stress tolerance. The scores are normally distributed, with a mean of 62 and a standard deviation of 8. If only the top 15% of recruits are selected, find the cutoff score.

Step 1: Draw the normal distribution curve.



Ex.) 9 – Applications of the Normal Distribution

Step 2: Subtract $1 - 0.1500$ to find area to the left, 0.8500 .
Look for the closest value to that in Table 3.

<i>z</i>	.00	.01	.02	.03	.04	.05
0.0	.5000	.5040	.5080	.5120	.5160	.5199
0.1	.5398	.5438	.5478	.5517	.5557	.5596
0.2	.5793	.5832	.5871	.5910	.5948	.5987
0.3	.6179	.6217	.6255	.6293	.6331	.6368
0.4	.6554	.6591	.6628	.6664	.6700	.6736
0.5	.6915	.6950	.6985	.7019	.7054	.7088
0.6	.7257	.7291	.7324	.7357	.7389	.7422
0.7	.7580	.7611	.7642	.7673	.7704	.7734
0.8	.7881	.7910	.7939	.7967	.7995	.8023
0.9	.8159	.8186	.8212	.8238	.8264	.8289
1.0	.8413	.8438	.8461	.8485	.8508	.8531
1.1	.8643	.8665	.8686	.8708	.8729	.8749
1.2	.8849	.8869	.8888	.8907	.8925	.8944

$z = 1.04$



Ex.) 9 – Applications of the Normal Distribution

Step 3: Find X .

If we rearrange the equation $z = \frac{X - \mu}{\sigma}$, solving for X :

$$X = \mu + z\sigma = 62 + 1.04(8) = 70.32$$

The cutoff, the lowest possible score to qualify, is 70.



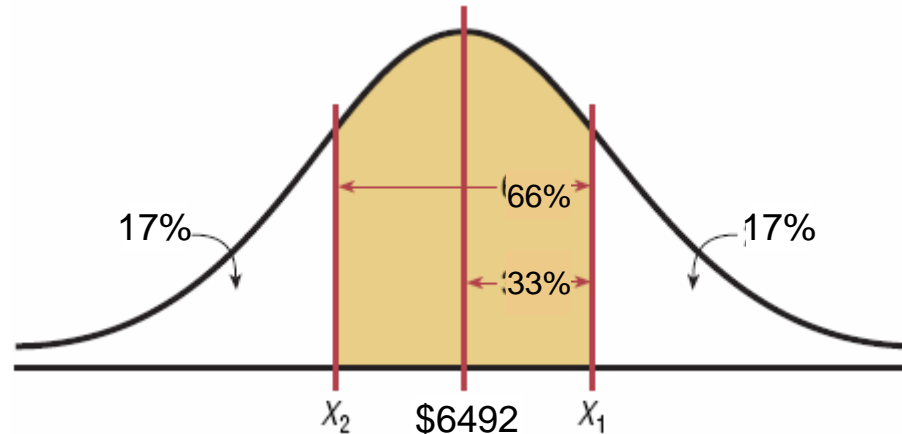
Applications of the Normal Distribution

- Students who score in the bottom 10% on a mathematics admission test at Greenville Tech are required to take a remedial math course. If the average score is 78 with a standard deviation of 6, what is the score that determines if a student does or does not take the remedial course?

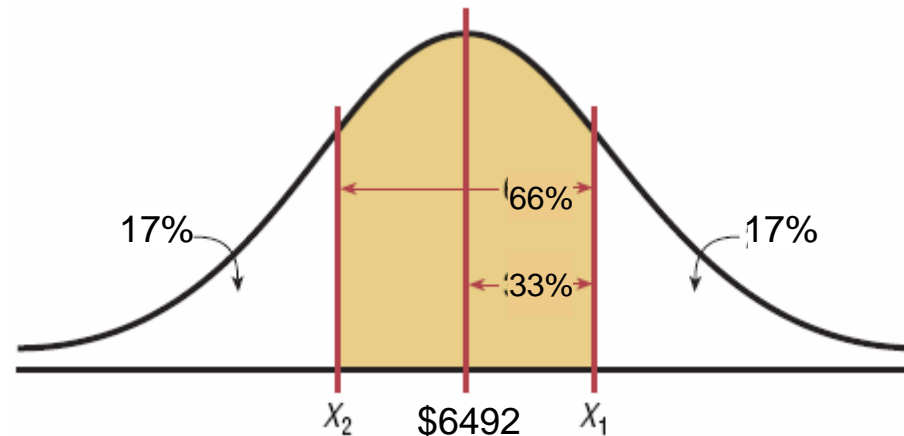
Ex.) 10 – Applications of the Normal Distribution

Used Boat Prices A marine sales dealer finds that the average price of a previously owned boat is \$6492. He decides to sell boats that will appeal to the middle 66% of the market in terms of price. Find the maximum and minimum prices of the boats the dealer will sell. The standard deviation is \$1025, and the variable is normally distributed.

Step 1: Draw the normal distribution curve.



Ex.) 10 – Applications of the Normal Distribution



Area to the left of the positive z : $0.5000 + 0.3300 = 0.8300$.

Using Table 3, $z \approx 0.95$. $X = 6492 + 0.95(1025) = 7465.75$

Area to the left of the negative z : $0.5000 - 0.3300 = 0.1700$.

Using Table 3, $z \approx -0.95$. $X = 6492 - 0.95(1025) = 5518.25$

The middle 66% of readings are between \$5518.25 and \$7465.75.



Applications of the Normal Distribution

- Papa Fred's Pizza has found that the mean time to deliver a pizza is 21.2 minutes with a standard deviation of 6.1 minutes. They want to have a guaranteed delivery time. In order to deliver 99% within the guaranteed time you need to find the time represented by the 99th percentile. What is this value?



Class Activity

- John finds that there is a scholarship available to all persons scoring in the top 5% on the ACT test. If the mean score on the ACT is 21 with a standard deviation of 4.7, what score does he need in order to qualify for the scholarship?



6-5 The Normal Approximation to the Binomial Distribution

A normal distribution is often used to solve problems that involve the binomial distribution since when n is large (say, 100), the calculations are too difficult to do by hand using the binomial distribution.



The Normal Approximation to the Binomial Distribution

- The normal approximation to the binomial is appropriate when $np \geq 5$ and $nq \geq 5$.
- In addition, a **correction for continuity** must be used in the normal approximation to the binomial.
- The continuity correction means that for any specific value of X , say 8, the boundaries of X in the binomial distribution (in this case, 7.5 to 8.5) must be used.



The Normal Approximation to the Binomial Distribution

Binomial	Normal
When finding:	Use:
$P(X = a)$	$P(a - 0.5 < X < a + 0.5)$
$P(X \geq a)$	$P(X > a - 0.5)$
$P(X > a)$	$P(X > a + 0.5)$
$P(X \leq a)$	$P(X < a + 0.5)$
$P(X < a)$	$P(X < a - 0.5)$
$P(a < X < b)$	$P(a + 0.5 < X < b - 0.5)$
$P(a \leq X \leq b)$	$P(a - 0.5 < X < b + 0.5)$

For all cases, $\mu = np$, $\sigma = \sqrt{npq}$, $np \geq 5$, $nq \geq 5$



The Normal Approximation to the Binomial Distribution

Procedure Table

Step 1: Check to see whether the normal approximation can be used.


Step 2: Find the mean μ and the standard deviation σ .

Step 3: Write the problem in probability notation, using X .

Step 4: Rewrite the problem by using the continuity correction factor, and show the corresponding area under the normal distribution.

Step 5: Find the corresponding z values.

Step 6: Find the solution.



Ex.) 11 – Normal Approximation to the Binomial Distribution

Youth Smoking Two out of five adult smokers acquired the habit by age 14. If 400 smokers are randomly selected, find the probability that 170 or more acquired the habit by age 14.

Here, $p = 0.4$, $q = 0.6$, and $n = 400$.

Step 1: Check to see whether a normal approximation can be used.


$$np = (400)(0.4) = 160 \text{ and } nq = (400)(0.6) = 240$$

Since $np \geq 5$ and $nq \geq 5$, we can use the normal distribution.

Step 2: Find the mean and standard deviation.

$$\mu = np = (400)(0.4) = 160$$

$$\sigma = \sqrt{npq} = \sqrt{400(0.4)(0.6)} = 9.80$$



Ex.) 11 – Normal Approximation to the Binomial Distribution

Step 3: Write in probability notation. $P(X \geq 170)$

Step 4: Rewrite using the continuity correction factor.

$$P(X > 169.5)$$

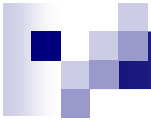
Step 5: Find the corresponding z value.

$$z = \frac{169.5 - 160}{9.8} = 0.97$$

Step 6: Find the solution

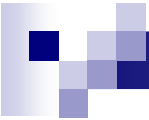
The area greater than the z value is
 $1 - 0.8340 = 0.1660$, or 16.6%.

Hence, the probability that more than 170 smokers had acquired the habit of smoking by age 14 is 16.6%.




Normal Approximation to the Binomial Distribution

- Suppose that 30% of the vehicles in a mall parking lot belong to employees. 50 vehicles are chosen at random. Find the probability that:
 - **At most 3** belong to mall employees.
 - **At least 7** belong to mall employees.



Normal Approximation to the Binomial Distribution

- A survey indicates that 23% of US men select fishing as their favorite leisure activity.
 - If you randomly select 100 men, find the probability that exactly two of the men liked fishing.
 - Again, if you randomly select 100 men, find the probability that at least 2 of the men liked fishing.



Class Activity: Normal Approximation to the Binomial Distribution

Recall that for use of a normal distribution as an approximation to the binomial distribution, the conditions $np \geq 5$ and $nq \geq 5$ must be met. For each given probability, compute the minimum sample size needed for use of the normal approximation.

a. $p = 0.1$

b. $p = 0.3$

c. $p = 0.5$

d. $p = 0.8$

e. $p = 0.9$